

Questions related to SECTION 4.8

1. Determine the following indefinite integrals.

(a) $\int (3x^5 - 5x^9) dx$

$$\int (3x^5 - 5x^9) dx = \frac{3x^6}{6} - \frac{5x^{10}}{10} + C = \frac{x^6}{2} - \frac{x^{10}}{2} + C$$

(b) $\int (3u^{-2} - 4u^2 + 1) du$

$$\int (3u^{-2} - 4u^2 + 1) du = \frac{3u^{-1}}{-1} - \frac{4u^3}{3} + u + C = -\frac{3}{u} - \frac{4u^3}{3} + u + C$$

(c) $\int 6\sqrt[3]{x} dx$

$$\int 6\sqrt[3]{x} dx = \int 6x^{1/3} dx = \frac{6x^{4/3}}{\frac{4}{3}} + C = 6\frac{3}{4}x^{4/3} + C = \frac{9}{2}x^{4/3} + C$$

(d) $\int (\sin 2y + \cos 3y) dy$

$$\int (\sin 2y + \cos 3y) dy = -\frac{\cos 2y}{2} + \frac{\sin 3y}{3} + C$$

(e) $\int (\sec^2(\theta) + \sec \theta \tan \theta) d\theta$

$$\int (\sec^2(\theta) + \sec \theta \tan \theta) d\theta = \tan \theta + \sec \theta + C$$

(f) $\int \frac{6}{\sqrt{25 - x^2}} dx$

$$\int \frac{6}{\sqrt{25 - x^2}} dx = 6 \sin^{-1} \left(\frac{x}{5} \right) + C$$

see Briggs Calculus p. 297, table 4.6, rule 9

(g) $\int \frac{2}{16z^2 + 25} dz$

$$\begin{aligned} \int \frac{2}{16z^2 + 25} dz &= 2 \int \frac{1}{16 \left(z^2 + \frac{25}{16} \right)} dz = \frac{1}{8} \int \frac{1}{z^2 + \left(\frac{5}{4} \right)^2} dz = \\ &= \frac{1}{8} \cdot \frac{4}{5} \tan^{-1} \left(\frac{4}{5} z \right) + C = \frac{1}{10} \tan^{-1} \left(\frac{4}{5} z \right) + C \end{aligned}$$

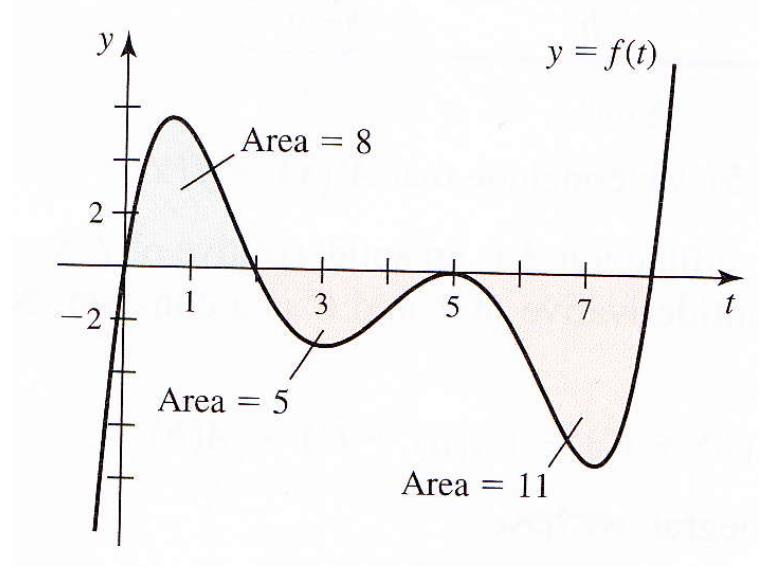
see Briggs Calculus, p. 297, table 4.6, rule 10

Questions related to SECTION 5.3

1. The graph of f is shown in the figure. Let

$$A(x) = \int_0^x f(t) dt \quad \text{and} \quad F(x) = \int_2^x f(t) dt$$

be two area functions of f . Evaluate the following area functions.



- (a) $A(2) = \int_0^2 f(t) dt = 8$
 (b) $F(5) = \int_2^5 f(t) dt = -5$
 (c) $A(0) = \int_0^0 f(t) dt = 0$
 (d) $A(8) = \int_0^8 f(t) dt = 8 - 5 - 11 = -8$
 (e) $F(8) = \int_2^8 f(t) dt = -5 - 11 = -16$
 (f) $A(8) = \int_0^5 f(t) dt = 8 - 5 = 3$

2. Evaluate the following integrals using the Fundamental Theorem of Calculus.

(a) $\int_{-2}^2 (x^2 - 4) dx$

$$\int_{-2}^2 (x^2 - 4) dx = \left(\frac{x^3}{3} - 4x \right) \Big|_{-2}^2 = \frac{8}{3} - 8 + \frac{8}{3} - 8 = \frac{16}{3} - 16 = -\frac{32}{3}$$

(b) $\int_{\frac{1}{2}}^1 (x^{-3} - 8) dx$

$$\int_{\frac{1}{2}}^1 (x^{-3} - 8) dx = \left(\frac{x^{-2}}{-2} - 8x \right) \Big|_{\frac{1}{2}}^1 = -\frac{1}{2} - 8 + 2 + 4 = -\frac{5}{2}$$

(c) $\int_0^{\frac{\pi}{4}} \sec^2(\theta) d\theta$

$$\int_0^{\frac{\pi}{4}} \sec^2(\theta) d\theta = (\tan \theta) \Big|_0^{\frac{\pi}{4}} = \tan\left(\frac{\pi}{4}\right) - \tan(0) = 1 - 0 = 1$$

(d) $\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = (\sin^{-1} x) \Big|_0^{\frac{1}{2}} = \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0) = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

(e) $\int_1^2 \frac{3}{t} dt$

$$\int_1^2 \frac{3}{t} dt = (3 \ln |t|) \Big|_1^2 = 3 \ln(2) - 3 \ln(1) = 3 \ln(2) = \ln(8)$$

(f) $\int_4^9 \frac{x - \sqrt{x}}{x^3} dx$

$$\begin{aligned} \int_4^9 \frac{x - \sqrt{x}}{x^3} dx &= \int_4^9 \left(\frac{1}{x^2} - \frac{1}{x^{\frac{5}{2}}} \right) dx = \int_4^9 \left(x^{-2} - x^{-\frac{5}{2}} \right) dx = \\ &= \left(\frac{x^{-1}}{-1} - \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} \right) \Big|_4^9 = \left(-\frac{1}{x} + \frac{2}{3x^{\frac{3}{2}}} \right) \Big|_4^9 = -\frac{1}{9} + \frac{2}{81} + \frac{1}{4} - \frac{2}{24} = \frac{13}{162} \end{aligned}$$

3. Simplify the following expressions.

(a) $\frac{d}{dx} \int_3^x (t^2 + t + 1) dt$

$$\frac{d}{dx} \int_3^x (t^2 + t + 1) dt = x^2 + x + 1$$

(b) $\frac{d}{dx} \int_{x^2}^{10} \frac{dz}{z^2 + 1}$

$$\frac{d}{dx} \int_{x^2}^{10} \frac{dz}{z^2 + 1} = -\frac{d}{dx} \int_{10}^{x^2} \frac{dz}{z^2 + 1} = -\frac{1}{x^4 + 1} 2x = -\frac{2x}{x^4 + 1}$$

$$(c) \frac{d}{dx} \int_a^{\tan^{-1} x} \cos(\ln t)^{\tan t} dt$$

$$\frac{d}{dx} \int_a^{\tan^{-1} x} \cos(\ln t)^{\tan t} dt = \cos(\ln(\tan^{-1} x))^{\tan(\tan^{-1} x)} \frac{1}{1+x^2} = \frac{\cos(\ln(\tan^{-1} x))^x}{1+x^2}$$

Questions related to SECTION 5.4

1. Use symmetry to evaluate the following integrals.

$$(a) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x dx$$

Because $\cos(-x) = \cos(x)$, $\cos(x)$ function is even. Therefore the integral can be calculated as:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x dx = 2 \int_0^{\frac{\pi}{4}} \cos x dx = 2(\sin(x)) \Big|_0^{\frac{\pi}{4}} = 2 \left(\frac{\sqrt{2}}{2} - 0 \right) = \sqrt{2}$$

$$(b) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^5(x) dx$$

Because $\sin(-x) = -\sin(x)$, the $\sin x$ function is odd. Therefore the integral can be calculated as:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^5(x) dx = 0$$

Questions related to SECTION 5.5

1. Find the following integrals using substitution rule.

$$(a) \int 2x(x^2 + 1)^4 dx$$

Let

$$u = x^2 + 1 \quad \Rightarrow \quad \frac{du}{dx} = 2x \quad \Rightarrow \quad dx = \frac{du}{2x}$$

$$\int 2x(x^2 + 1)^4 dx = \int 2x u^4 \frac{du}{2x} = \int u^4 du = \frac{u^5}{5} + C = \frac{(x^2 + 1)^5}{5} + C$$

(b) $\int \sin^3(x) \cos x \, dx$

Let

$$u = \sin(x) \Rightarrow \frac{du}{dx} = \cos x \Rightarrow dx = \frac{du}{\cos x}$$

$$\int \sin^3(x) \cos x \, dx = \int u^3 \cos x \frac{du}{\cos x} = \int u^3 \, du = \frac{u^4}{4} + C = \frac{\sin^4(x)}{4} + C$$

(c) $\int (x^2 + x)^{10} (2x + 1) \, dx$

Let

$$u = x^2 + x \Rightarrow \frac{du}{dx} = 2x + 1 \Rightarrow dx = \frac{du}{2x + 1}$$

$$\int (x^2 + x)^{10} (2x + 1) \, dx = \int u^{10} (2x + 1) \frac{du}{2x + 1} = \int u^{10} \, du = \frac{u^{11}}{11} + C = \frac{(x^2 + x)^{11}}{11} + C$$

(d) $\int \frac{1}{\sqrt{1 - 9x^2}} \, dx$

$$\int \frac{1}{\sqrt{1 - 9x^2}} \, dx = \int \frac{1}{\sqrt{1 - (3x)^2}} \, dx$$

Let

$$u = 3x \Rightarrow \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$$

$$\frac{1}{3} \int \frac{1}{\sqrt{1 - (u)^2}} \, du = \frac{1}{3} \sin^{-1}(u) + C = \frac{\sin^{-1}(3x)}{3} + C$$

(e) $\int x^9 \sin x^{10} \, dx$

Let

$$u = x^{10} \Rightarrow \frac{du}{dx} = 10x^9 \Rightarrow dx = \frac{du}{10x^9}$$

$$\int x^9 \sin x^{10} \, dx = \int x^9 \sin u \frac{du}{10x^9} = \frac{1}{10} \int \sin u \, du = -\frac{1}{10} \cos u + C = -\frac{1}{10} \cos x^{10} + C$$

(f) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$

Let

$$u = e^x + e^{-x} \Rightarrow \frac{du}{dx} = e^x - e^{-x} \Rightarrow dx = \frac{du}{e^x - e^{-x}}$$

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx = \int \frac{e^x - e^{-x}}{u} \frac{du}{e^x - e^{-x}} \, dx = \int \frac{1}{u} \, du = \ln |u| + C = \ln |e^x + e^{-x}| + C$$

$$(g) \int_0^2 \frac{2x}{(x^2 + 1)^2} dx$$

Let

$$u = x^2 + 1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

$$\int_0^2 \frac{2x}{(x^2 + 1)^2} dx = \int_{x=0}^{x=2} \frac{2x du}{u^2 2x} = \int_{u=1}^{u=5} u^{-2} du = \left(\frac{u^{-1}}{-1} \right) \Big|_1^5 = -\frac{1}{5} + 1 = \frac{4}{5}$$

$$(h) \int_0^{\frac{\pi}{2}} \sin^2(\theta) \cos \theta d\theta$$

Let

$$u = \sin \theta \Rightarrow \frac{du}{d\theta} = \cos \theta \Rightarrow d\theta = \frac{du}{\cos \theta}$$

$$\int_0^{\frac{\pi}{2}} \sin^2(\theta) \cos \theta d\theta = \int_{\theta=0}^{\theta=\frac{\pi}{2}} u^2 \cos \theta \frac{du}{\cos \theta} = \int_{u=0}^{u=1} u^2 du = \left(\frac{u^3}{3} \right) \Big|_0^1 = \frac{1}{3}$$

$$(i) \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2(x)} dx$$

Let

$$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$$

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2(x)} dx &= \int_{x=0}^{x=\frac{\pi}{4}} \frac{\sin x}{u^2} \frac{du}{-\sin x} = - \int_{u=1}^{u=\frac{\sqrt{2}}{2}} u^{-2} du = - \left(\frac{u^{-1}}{-1} \right) \Big|_1^{\frac{\sqrt{2}}{2}} = \\ &= \frac{2}{\sqrt{2}} - 1 = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1 \end{aligned}$$

$$(j) \int_{-1}^2 x^2 e^{x^3+1} dx$$

Let

$$u = x^3 + 1 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$$

$$\int_{-1}^2 x^2 e^{x^3+1} dx = \int_{x=-1}^{x=2} x^2 e^u \frac{du}{3x^2} = \frac{1}{3} \int_{u=0}^{u=9} e^u du = \frac{e^u}{3} \Big|_0^9 = \frac{e^9 - 1}{3}$$

$$(k) \int_0^4 \frac{p}{\sqrt{9+p^2}} dp$$

Let

$$u = 9 + p^2 \Rightarrow \frac{du}{dp} = 2p \Rightarrow dp = \frac{du}{2p}$$

$$\int_0^4 \frac{p}{\sqrt{9+p^2}} dp = \int_{p=0}^{p=4} \frac{p}{\sqrt{u}} \frac{du}{2p} = \frac{1}{2} \int_{u=9}^{u=25} \frac{1}{\sqrt{u}} du = \frac{1}{2} (2\sqrt{u}) \Big|_9^{25} = \sqrt{25} - \sqrt{9} = 2$$