Faculty of Engineering

DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING

EENG428 Introduction to Robotics

Instructor:

M. K. Uyguroğlu

Midterm EXAMINATION

April 21, 2014

Duration : 100 minutes

Number of Problems: 5

Good Luck

STUDENT'S	
NUMBER	
NAME	SOLUTIONS
SURNAME	

Problem		Points
1		15
2		15
3		20
4		25
5		25
TOTAL		100

Problem 1

For frame F , find the values of the missing elements and complete the matrix representation of the frame.

$$F = \begin{bmatrix} 0.433 & b & 0.75 & 0 \\ 0.25 & 0.866 & c & 0 \\ a & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In order to determine the $a, b,$ and c we will use the following relations:

$$n.o = o.a = n.a = 0$$

$$0.433 \times b + 0.25 \times 0.866 + a \times 0 = 0$$

$$b = -\frac{0.25 \times 0.866}{0.433} = -0.5$$

$$0.75 \times b + c \times 0.866 + 0.5 \times 0 = 0$$

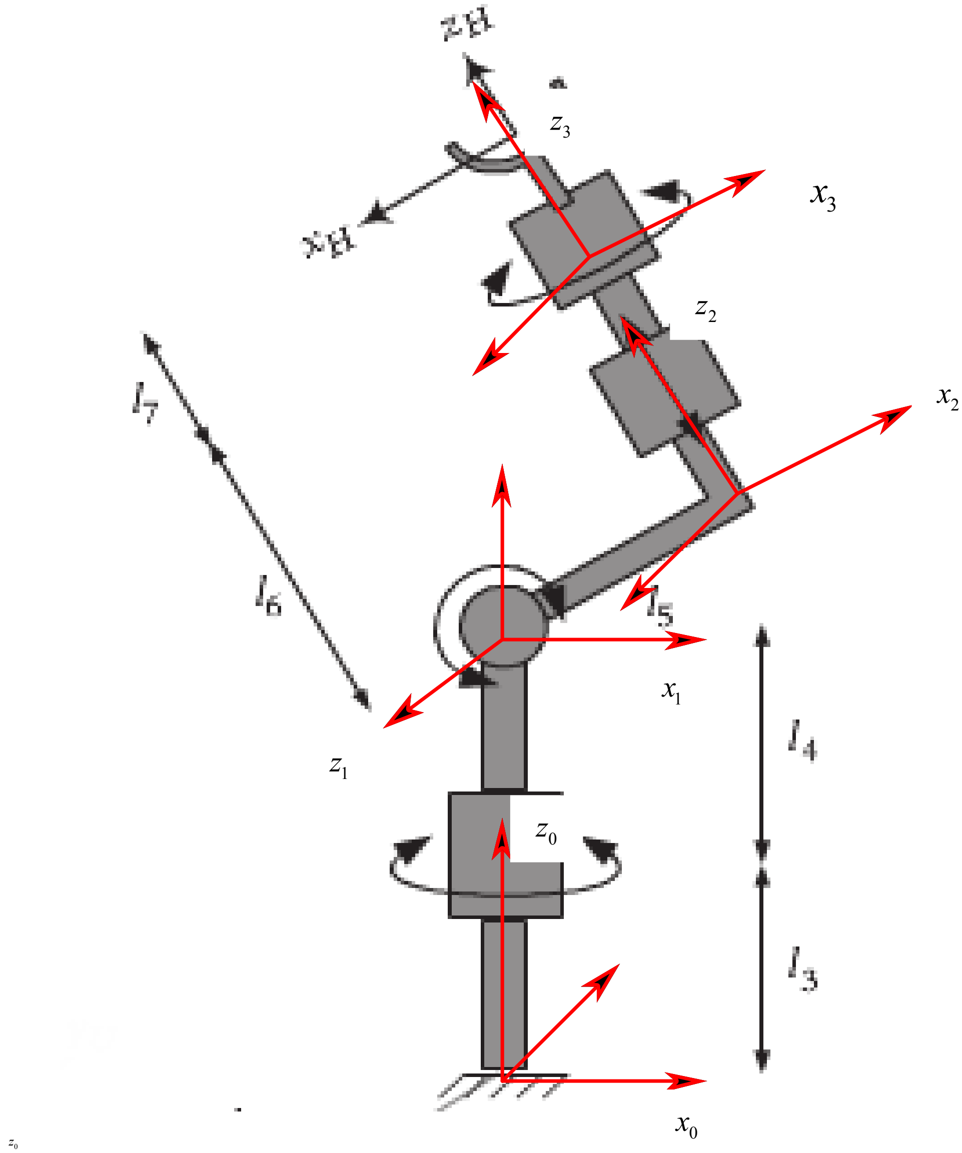
$$c = \frac{0.75 \times 0.5}{0.866} = 0.433$$

$$0.433 \times 0.75 + 0.25 \times c + a \times 0.5 = 0$$

$$a = \frac{-0.433 \times 0.75 - 0.25 \times 0.433}{0.5} = -0.866$$

Problem 2

For the given specialty designed 4-DOF robot:

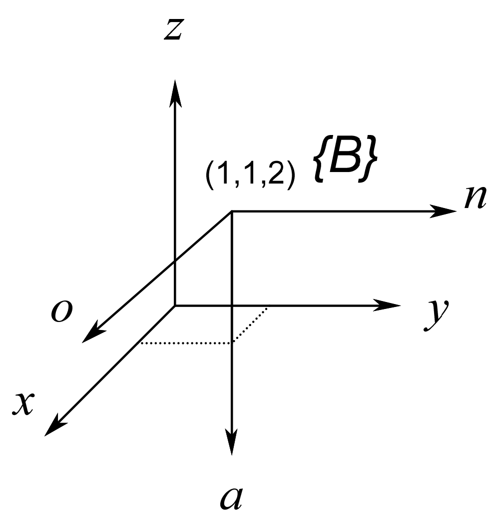


- Assign appropriate frames for the Denavit-Hartenberg representation.
- Fill out the parameters table.

Link	θ	d	a	α
1	θ_1	$\ell_3 + \ell_4$	0	90
2	θ_2	0	ℓ_5	-90
3	0	ℓ_6	0	0
4	θ_4	ℓ_7	0	0

Problem 3

A frame B shown below, is rotated 90° about the z-axis, then translated 3 and 5 units relative to the n- and o-axes respectively, then rotated another 90° about the n-axis,and finally, 90° about the y-axis. Find the new location and orientation of the frame.



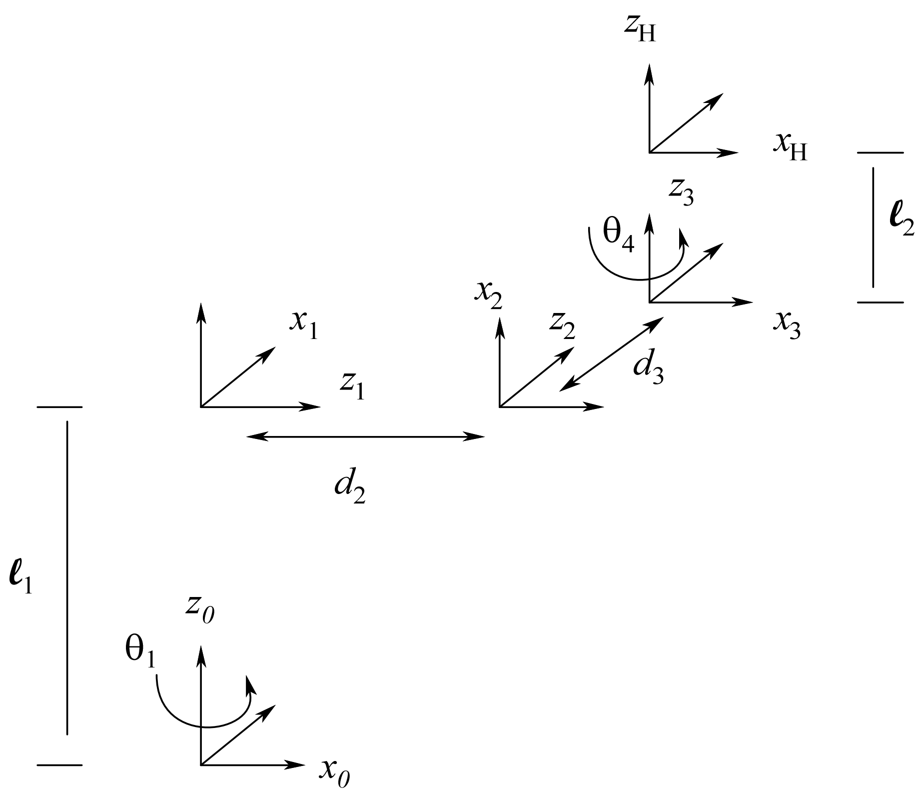
$$B' = Rot(y,90)Rot(z,90)BTrans(3,5,0)Rot(x,90)$$

$$B' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 0 & 0 & -1 & 6 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4

For a RPPR manipulator the frames which are assigned to the joints are shown below.



- Fill out the parameters table.

Link	θ	d	a	α
1	θ_1	ℓ_1	0	90
2	90	d_2	0	90
3	90	d_3	0	90
4	θ_4	ℓ_2	0	0

- Write all the A matrices.
- Write an equation in terms of A matrices that shows how ${}^0T_H$ can be calculated.

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T = A_1 A_2 A_3 A_4$

Problem 5

Suppose that a robot is made of a Cartesian and RPY combination of joints. Find the necessary RPY angles to achieve the following:

$$F = \begin{bmatrix} 0 & -0.866 & 0.5 & 3 \\ 0.5 & 0.433 & 0.75 & 1 \\ -0.866 & 0.25 & 0.433 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = Cart(P_x, P_y, P_z) \times RPY(\theta_z, \theta_y, \theta_x) = Trans(P_x, P_y, P_z) Rot(z, \theta_z) Rot(y, \theta_y) Rot(x, \theta_x)$$

$$F = \begin{bmatrix} C\theta_z C\theta_y & C\theta_z S\theta_y S\theta_x - C\theta_x S\theta_z & C\theta_x C\theta_z S\theta_y + S\theta_x S\theta_z & P_x \\ S\theta_z C\theta_y & S\theta_z S\theta_y S\theta_x + C\theta_x C\theta_z & C\theta_x S\theta_z S\theta_y - S\theta_x C\theta_z & P_y \\ -S\theta_y & S\theta_x C\theta_y & C\theta_x C\theta_y & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This implies that:

$$-\sin\theta_y = -0.866$$

$$\theta_y = a\tan 2\left(0.866, \mp\sqrt{1-0.866^2}\right)$$

$$\theta_{y_1} = 60$$

$$\theta_{y_2} = 120$$

$$\sin\theta_x \cos\theta_y = 0.25$$

$$\cos\theta_x \cos\theta_y = 0.433$$

$$\theta_{x_1} = a\tan 2\left(\frac{0.25}{\cos 60}, \frac{0.433}{\cos 60}\right) = 30^0$$

$$\theta_{x_2} = a\tan 2\left(\frac{0.25}{\cos 120}, \frac{0.433}{\cos 120}\right) = -150^0$$

$$\cos\theta_z \cos\theta_y = 0$$

$$\sin\theta_z \cos\theta_y = 0.5$$

$$\theta_{z_1} = a\tan 2\left(\frac{0.5}{\cos 60}, 0\right) = 90^0$$

$$\theta_{z_1} = a\tan 2\left(\frac{0.5}{\cos 120}, 0\right) = -90^0$$

GIVEN:

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sin(\alpha \mp \beta) = \sin\alpha \cos\beta \mp \sin\beta \cos\alpha$$

$$\cos(\alpha \mp \beta) = \cos\alpha \cos\beta \pm \sin\beta \sin\alpha$$

$$\text{if} \quad \cos\theta = b \quad \text{then} \quad \theta = A\tan 2(\pm\sqrt{1-b^2}, b)$$

$$\text{if} \quad \sin\theta = b \quad \text{then} \quad \theta = A\tan 2(b, \pm\sqrt{1-b^2})$$

$$\text{if} \quad a\cos\theta + b\sin\theta = c \quad \text{then} \quad \theta = A\tan 2(b,a) + A\tan 2(\pm\sqrt{a^2+b^2-c^2}, c)$$

$$\text{if} \quad a\cos\theta - b\sin\theta = 0 \quad \text{then} \quad \theta = A\tan 2(a,b) \quad \text{and} \quad \theta = A\tan 2(-a,-b)$$