

PHYS101 Midterm Exam – Solution Set

Department of Physics

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Questions:

1. Given $\vec{A} = 3\hat{i} - 4\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - 4\hat{j} - 8\hat{k}$

(a) Find the vector $\vec{D} = 2\vec{A} - \frac{1}{2}\vec{B}$ in unit vector notation. (3 P)

Solution:

$$\vec{D} = 2(3\hat{i} - 4\hat{j} + 4\hat{k}) - \frac{1}{2}(2\hat{i} - 4\hat{j} - 8\hat{k}) = 5\hat{i} - 6\hat{j} + 12\hat{k}$$

(b) Find the magnitude of the vector \vec{D} . (3 P)

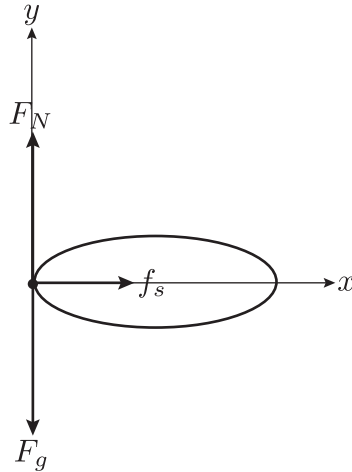
Solution:

$$D = |\vec{D}| = \sqrt{5^2 + (-6)^2 + (12)^2} = \sqrt{205} \approx 14.32$$

2. A bicycle enters a turn at a speed of 36km/h . The coefficient of static friction μ_s between the tires and the track is 0.40.

- (a) Draw the Free Body Diagram for the bicycle. (4 P)

Solution:



- (b) Identify the responsible force that makes this turn possible. (2 P)

Solution:

Static frictional force which is directed to the centre of the circle, as there is no motion in radial direction.

- (c) Calculate the minimum radius so that the bicycle still makes the turn. (3 P)

Solution:

From the free body diagram we get:

$$F_N - F_g = 0 \implies F_N = F_g = mg \quad \text{and} \quad f_s = \mu_s F_N = \mu_s mg$$

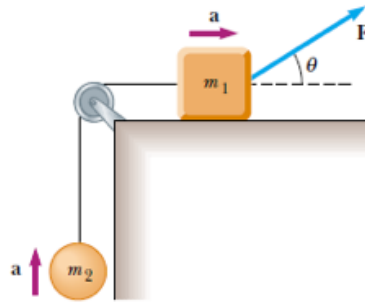
$$f_s = m \frac{v^2}{R}$$

$$\mu_s mg = m \frac{v^2}{R}$$

$$R = \frac{v^2}{\mu_s g}, \quad v = 36 \frac{\text{km}}{\text{h}} = 36 \frac{1000\text{m}}{3600\text{s}} = 10 \frac{\text{m}}{\text{s}}$$

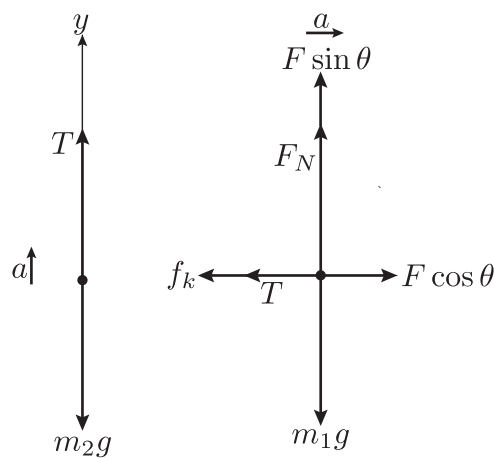
$$R = \frac{(10 \frac{\text{m}}{\text{s}})^2}{0.4 \cdot 9.8 \frac{\text{m}}{\text{s}}} = 25.5\text{m}$$

3. A block of mass $m_1 = 15\text{kg}$ and a ball of mass $m_2 = 8\text{kg}$ are connected by a massless string of over a massless pulley as shown in the figure below. The coefficient of kinetic friction between the block and the table is $\mu_k = 0.2$. A force of magnitude $F = 150\text{N}$ is applied at an angle of $\theta = 50^\circ$ as shown in the figure.



- (a) Draw the free body diagrams for m_1 and m_2 . (4 P)

Solution:



- (b) Calculate the acceleration of the two masses and the tension in the string. (6 P)

Solution:

From the free body diagram for the mass m_2 :

$$T - m_2g = m_2a \implies T = m_2(a + g) \quad (1)$$

From the free body diagram for m_1 :

$$\sum F_x = F \cos \theta - T - f_k = m_1a \quad (2)$$

$$\sum F_y = F_N + F \sin \theta - m_1g = 0 \quad (3)$$

From equation (3)

$$F_N = m_1g - F \sin \theta \implies f_k = \mu_k F_N = \mu_k(m_1g - F \sin \theta) \quad (4)$$

(4) and (1) in (2)

$$F \cos \theta - \mu_k(m_1g - F \sin \theta) - m_2(a + g) = m_1a \quad (5)$$

Solving (5) for a gives

$$a = \frac{F \cos \theta - \mu_k(m_1g - F \sin \theta) - m_2g}{m_1 + m_2} = \frac{150\text{N} \cos 50^\circ - 0.2(15\text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} - 150\text{N} \sin 50^\circ) - 8\text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2}}{15\text{kg} + 8\text{kg}} = 0.5 \frac{\text{m}}{\text{s}^2} \quad (6)$$

Now we can calculate the tension in the cord by substituting (6) in (1)

$$T = m_2(a + g) = 8\text{kg} \left(0.5 \frac{\text{m}}{\text{s}^2} + 9.8 \frac{\text{m}}{\text{s}^2} \right) = 82.4\text{N} \quad (7)$$

4. A particle moves in the xy -plane with the coordinates $x(t) = 2t - 1$ and $y(t) = 2t^2 + 3t$, with $x(t)$ and $y(t)$ in meters and t in seconds.

- (a) Write the position vector. (2 P)

Solution:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} = (2t - 1)\hat{i} + (2t^2 + 3t)\hat{j}$$

- (b) Write the initial position vector ($t = 0$). (1 P)

Solution:

$$\vec{r}(t = 0) = (-\hat{i})m$$

- (c) Find the displacement the particle between $t = 1s$ and $t = 5s$. (2 P)

Solution:

$$\begin{aligned}\vec{r}(t = 1s) &= (2 \cdot 1s - 1)\hat{i} + (2(1s)^2 + 3 \cdot 1s)\hat{j} = (\hat{i} + 5\hat{j})m \\ \vec{r}(t = 5s) &= (2 \cdot 5s - 1)\hat{i} + (2(5s)^2 + 3 \cdot 5s)\hat{j} = (9\hat{i} + 65\hat{j})m \\ \Delta\vec{r} &= \vec{r}(t = 5s) - \vec{r}(t = 1s) = (9\hat{i} + 65\hat{j})m - (\hat{i} + 5\hat{j})m = (8\hat{i} + 60\hat{j})m\end{aligned}$$

- (d) Find the average velocity between $t = 1s$ and $t = 5s$. (2 P)

Solution:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(8\hat{i} + 60\hat{j})m}{5s - 1s} = (2\hat{i} + 15\hat{j})\frac{m}{s}$$

- (e) Find velocity vector and the acceleration vector at any time t . (3 P)

Solution:

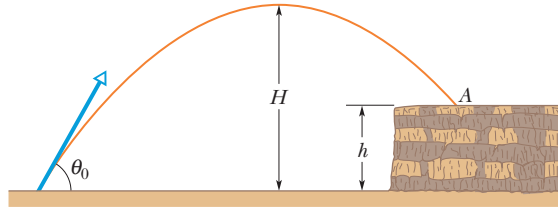
$$\begin{aligned}\vec{v}(t) &= (2\hat{i} + (4t + 3)\hat{j})\frac{m}{s} \\ \vec{a}(t) &= (4\hat{j})\frac{m}{s^2}\end{aligned}$$

- (f) Write the acceleration components a_x and a_y at $t = 3s$. (2 P)

Solution:

$$\vec{a}(t = 3s) = 4\hat{j} \implies a_x = 0 \text{ and } a_y = 4\frac{m}{s^2}$$

5. In the figure below, a stone is projected at a cliff of height h with an initial speed of 42.0m/s directed at angle $\theta_0 = 60.0^\circ$ above the horizontal. The stone strikes at A, 5.50s after launching.



Find

- (a) the height h of the cliff, (3 P)

Solution:

The height of the cliff is y_f of the accelerated motion in y -direction. As depicted in the figure above we put the origin of the coordinate system to the point where the stone is thrown. The direction of the positive y axis is pointing upward, and the direction of the positive x -axis is pointing to the right.

$$\begin{aligned} h &= y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2 = \\ &= 0 + 42.0\frac{m}{s} \sin(60^\circ) 5.50s - \frac{1}{2}9.80\frac{m}{s^2}(5.50s)^2 = 51.8m \end{aligned}$$

- (b) the speed of the stone just before impact at A, and (3 P)

Solution:

Before we determine the speed, let us determine the velocity in component notation at the moment of the impact.

$$\begin{aligned} v_x(t) &= v_{x0} = 42.0\frac{m}{s} \cos(60^\circ) = 21.0\frac{m}{s} \\ v_y(t) &= v_{y0} - gt = 42.0\frac{m}{s} \sin(60^\circ) - 9.80\frac{m}{s^2}5.50s = -17.5\frac{m}{s} \end{aligned}$$

Then we get the speed as the magnitude of the velocity vector as

$$v = \sqrt{v_x(t)^2 + v_y(t)^2} = \sqrt{\left(21.0\frac{m}{s}\right)^2 + \left(-17.5\frac{m}{s}\right)^2} = 27.3\frac{m}{s}$$

- (c) the maximum height H reached above the ground. (3 P)

Solution:

At the highest point of the trajectory we have $v_y = 0$ and $y = H$:

$$H = \frac{v_{y0}^2}{2g} = \frac{v_0^2 \sin^2 \theta}{2g} = \frac{\left(42.0\frac{m}{s} \sin(60^\circ)\right)^2}{2 \cdot 9.80\frac{m}{s^2}} = 67.5m$$

or

At the highest point the y -component of the velocity $v_y = 0$, therefore

$$v_y = 0 = v_0 \sin \theta - gt_{max} \implies t_{max} = \frac{v_0 \sin \theta}{g} = \frac{42\frac{m}{s} \sin 60^\circ}{9.8\frac{m}{s^2}} = 37s.$$

Then we get for the maximum height

$$H = y(t = 37s) = y_0 + v_{y0}t_{max} - \frac{1}{2}gt_{max}^2 = 0 + 42\frac{m}{s} \sin 60^\circ \cdot 37s - \frac{1}{2}9.8\frac{m}{s^2}(37s)^2 = 67.5m$$