PHYS101 Midterm Exam – Solution Set

Department of Physics

Fall 2014/15 - November 28, 2014

Questions:

1. Given $\vec{A} = 3\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ and $\vec{B} = 2\hat{\imath} - 4\hat{\jmath} - 8\hat{k}$

   (a) Find the vector $\vec{D} = 2\vec{A} - \frac{1}{2}\vec{B}$ in unit vector notation. (3 P)
   Solution:

   $$\vec{D} = 2(3\hat{\imath} - 4\hat{\jmath} + 4\hat{k}) - \frac{1}{2}(2\hat{\imath} - 4\hat{\jmath} - 8\hat{k}) = 5\hat{\imath} - 6\hat{\jmath} + 12\hat{k}$$

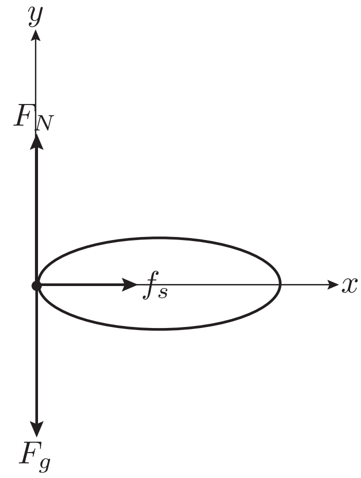
   (b) Find the magnitude of the vector $\vec{D}$. (3 P)
   Solution:

   $$D = \left|\vec{D}\right| = \sqrt{5^2 + (-6)^2 + (12)^2} = \sqrt{205} \approx 14.32$$

2. A bicycle enters a turn at a speed of $36km/h$. The coefficient of static friction μ_s between the tires and the track is 0.40.

(a) Draw the Free Body Diagram for the bicycle. (4 P)

Solution:



(b) Identify the responsible force that makes this turn possible. (2 P)

Solution:

Static frictional force which is directed to the centre of the circle, as there is no motion in radial direction.

(c) Calculate the minimum radius so that the bicycle still makes the turn. (3 P)

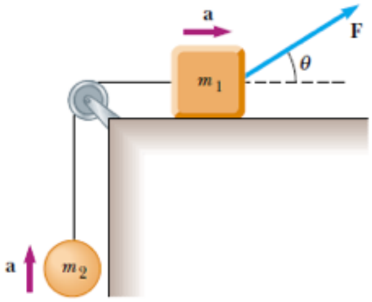
Solution:

From the free body diagram we get:

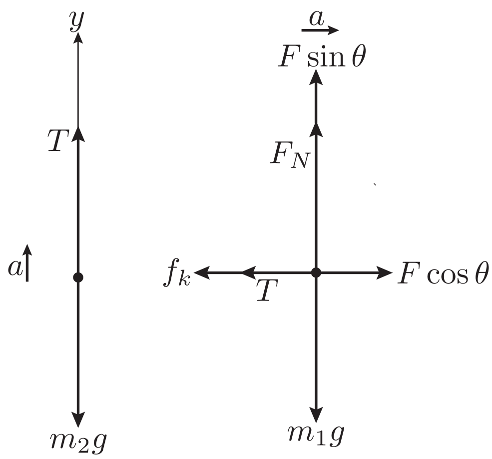
$$F_N - F_g = 0 \Longrightarrow F_N = F_g = mg \qquad \text{and } f_s = \mu_s F_N = \mu_s mg$$

$$\begin{aligned}
f_s &= m\frac{v^2}{R} \\
\mu_s mg &= m\frac{v^2}{R} \\
R &= \frac{v^2}{\mu_s g}, \qquad v = 36\frac{km}{h} = 36\frac{1000m}{3600s} = 10\frac{m}{s} \\
R &= \frac{\left(10\frac{m}{s}\right)^2}{0.4 \cdot 9.8\frac{m}{s}} = 25.5m
\end{aligned}$$

3. A block of mass $m_1 = 15kg$ and a ball of mass $m_2 = 8kg$ are connected by a massless string of over a massless pulley as shown in the figure below. The coefficient of kinetic friction between the block and the table is $\mu_k = 0.2$. A force of magnitude $F = 150N$ is applied at an angle of $\theta = 50°$ as shown in the figure.



(a) Draw the free body diagrams for m_1 and m_2. (4 P)
Solution:



(b) Calculate the acceleration of the two masses and the tension in the string. (6 P)
Solution:
From the free body diagram for the mass m_2:

$$T - m_2 g = m_2 a \Longrightarrow T = m_2(a+g) \tag{1}$$

From the free body diagram for m_1:

$$\sum F_x = F\cos\theta - T - f_k = m_1 a \tag{2}$$
$$\sum F_y = F_N + F\sin\theta - m_1 g = 0 \tag{3}$$

From equation (3)

$$F_N = m_1 g - F\sin\theta \Longrightarrow f_k = \mu_k F_N = \mu_k(m_1 g - F\sin\theta) \tag{4}$$

(4) and (1) in (2)

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a \tag{5}$$

Solving (5) for a gives

$$a = \frac{F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2 g}{m_1 + m_2} =$$
$$= \frac{150N\cos 50° - 0.2(15kg \cdot 9.8\frac{m}{s^2} - 150N\sin 50°) - 8kg \cdot 9.8\frac{m}{s^2}}{15kg + 8kg} = 0.5\frac{m}{s^2} \tag{6}$$

Now we can calculate the tension in the cord by substituting (6) in (1)

$$T = m_2(a+g) = 8kg\left(0.5\frac{m}{s^2} + 9.8\frac{m}{s^2}\right) = 82.4N \tag{7}$$

4. A particle moves in the xy-plane with the coordinates $x(t) = 2t - 1$ and $y(t) = 2t^2 + 3t$, with $x(t)$ and $y(t)$ in meters and t in seconds.

(a) Write the position vector. (2 P)
Solution:

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} = (2t - 1)\hat{\imath} + (2t^2 + 3t)\hat{\jmath}$$

(b) Write the initial position vector ($t = 0$). (1 P)
Solution:

$$\vec{r}(t = 0) = (-\hat{\imath})m$$

(c) Find the displacement the particle between $t = 1s$ and $t = 5s$. (2 P)
Solution:

$$\begin{aligned}
\vec{r}(t = 1s) &= (2 \cdot 1s - 1)\hat{\imath} + (2(1s)^2 + 3 \cdot 1s)\hat{\jmath} = (\hat{\imath} + 5\hat{\jmath})m \\
\vec{r}(t = 5s) &= (2 \cdot 5s - 1)\hat{\imath} + (2(5s)^2 + 3 \cdot 5s)\hat{\jmath} = (9\hat{\imath} + 65\hat{\jmath})m \\
\Delta\vec{r} &= \vec{r}(t = 5s) - \vec{r}(t = 1s) = (9\hat{\imath} + 65\hat{\jmath})m - (\hat{\imath} + 5\hat{\jmath})m = (8\hat{\imath} + 60\hat{\jmath})m
\end{aligned}$$

(d) Find the average velocity between $t = 1s$ and $t = 5s$. (2 P)
Solution:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(8\hat{\imath} + 60\hat{\jmath})m}{5s - 1s} = (2\hat{\imath} + 15\hat{\jmath})\frac{m}{s}$$

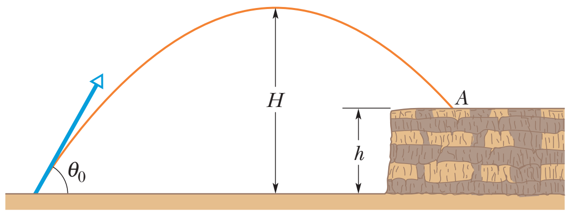
(e) Find velocity vector and the acceleration vector at any time t. (3 P)
Solution:

$$\begin{aligned}
\vec{v}(t) &= \left(2\hat{\imath} + (4t + 3)\hat{\jmath}\right)\frac{m}{s} \\
\vec{a}(t) &= \left(4\hat{\jmath}\right)\frac{m}{s^2}
\end{aligned}$$

(f) Write the acceleration components a_x and a_y at $t = 3s$. (2 P)
Solution:

$$\vec{a}(t = 3s) = 4\hat{\jmath} \Longrightarrow a_x = 0 \text{ and } a_y = 4\frac{m}{s^2}$$

5. In the figure below, a stone is projected at a cliff of height h with an initial speed of $42.0m/s$ directed at angle $\theta_0 = 60.0^\circ$ above the horizontal. The stone strikes at A, $5.50s$ after launching.



Find

(a) the height h of the cliff, (3 P)
Solution:
The height of the cliff is y_f of the accelerated motion in y-direction. As depicted in the figure above we put the origin of the coordinate system to the point where the stone is thrown. The direction of the positive y axis is pointing upward, and the direction of the positive x-axis is pointing to the right.

$$\begin{aligned} h &= y(t) = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = \\ &= 0 + 42.0\frac{m}{s} \sin(60^\circ)\, 5.50s - \frac{1}{2} 9.80\frac{m}{s^2}(5.50s)^2 = 51.8m \end{aligned}$$

(b) the speed of the stone just before impact at A, and (3 P)
Solution:
Before we determine the speed, let us determine the velocity in component notation at the moment of the impact.

$$\begin{aligned} v_x(t) &= v_{x_0} = 42.0\frac{m}{s} \cos(60^\circ) = 21.0\frac{m}{s} \\ v_y(t) &= v_{y_0} - gt = 42.0\frac{m}{s} \sin(60^\circ) - 9.80\frac{m}{s^2} 5.50s = -17.5\frac{m}{s} \end{aligned}$$

Then we get the speed as the magnitude of the velocity vector as

$$v = \sqrt{v_x(t)^2 + v_y(t)^2} = \sqrt{\left(21.0\frac{m}{s}\right)^2 + \left(-17.5\frac{m}{s}\right)^2} = 27.3\frac{m}{s}$$

(c) the maximum height H reached above the ground. (3 P)
Solution:
At the highest point of the trajectory we have $v_y = 0$ and $y = H$:

$$H = \frac{v_{y_0}^2}{2g} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{\left(42.0\frac{m}{s}\sin(60^\circ)\right)^2}{2 \cdot 9.80\frac{m}{s^2}} = 67.5m$$

or
At the highest point the y-component of the velocity $v_y = 0$, therefore

$$v_y = 0 = v_0 \sin\theta - g t_{max} \Longrightarrow t_{max} = \frac{v_0 \sin\theta}{g} = \frac{42\frac{m}{s}\sin 60^\circ}{9.8\frac{m}{s^2}} = 37s.$$

Then we get for the maximum height

$$H = y(t = 37s) = y_0 + v_{y_0} t_{max} - \frac{1}{2} g t_{max} = 0 + 42\frac{m}{s} \sin 60^\circ \cdot 37s - \frac{1}{2}\, 9.8\frac{m}{s^2}(37s)^2 = 67.5m$$