

PHYS101 Midterm Exam - Solution Set

Department of Physics

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Questions:

1. Given $\vec{A} = (4\hat{i} + 6\hat{j})m$ and $\vec{B} = (-3\hat{i} + 7\hat{j})m$

(a) Find the vector $\vec{D} = \frac{1}{2}\vec{A} + 2\vec{B}$ in unit vector notation. (2 P)

Solution:

$$\vec{D} = \frac{1}{2}(4\hat{i} + 6\hat{j})m + 2(-3\hat{i} + 7\hat{j})m = (-4\hat{i} + 17\hat{j})m$$

(b) Find the magnitude of the vector \vec{D} . (2 P)

Solution:

$$|\vec{D}| = \sqrt{(-4m)^2 + (17m)^2} = 17.46m$$

(c) Find the angle the vector \vec{D} makes with the positive x -axis. (2 P)

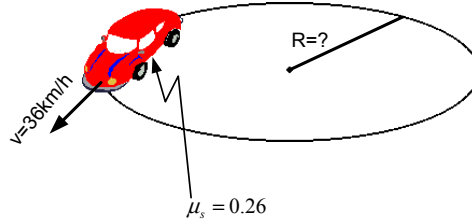
Solution:

$$\theta = \tan^{-1}\left(\frac{17}{-4}\right) = -76.76^\circ$$

As the vector is the 2nd quadrant the angle becomes

$$\theta = -76.76^\circ + 180^\circ = 103.24^\circ$$

2. What is the smallest radius of an unbanked (flat) track around which a car can travel at a speed of 36km/h and the coefficient of static friction between the tires and the road is $\mu_s = 0.26$? (5P)



Solution:

$$f_s = mg\mu_s, F_c = m\frac{v^2}{r}$$

The centripetal force must be equal to the static frictional force

$$mg\mu_s = m\frac{v^2}{r} \iff r = \frac{v^2}{\mu_s g} = \frac{(10\text{m/s})^2}{0.26 \times 9.8\frac{\text{m}}{\text{s}^2}} = 39.24\text{m}$$

3. A particle moves in the xy -plane with the coordinates $x(t) = -2t + 3$ and $y(t) = -3t^2 - 4t$, with $x(t)$ and $y(t)$ in meters and t in seconds.

- (a) Write the position vector. (1 P)

Solution:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} = (-2t + 3)\hat{i} + (-3t^2 - 4t)\hat{j}$$

- (b) Find the displacement the particle between $t = 1\text{s}$ and $t = 5\text{s}$. (2 P)

Solution:

$$\begin{aligned} \Delta\vec{r} = \vec{r}(5\text{s}) - \vec{r}(1\text{s}) &= [(-2 \times 5 + 3)\hat{i} + (-3 \times 5^2 - 4 \times 5)]\text{m} - \\ &- [(-2 \times 1 + 3)\hat{i} + (-3 \times 1^2 - 4 \times 1)]\text{m} = (-8\hat{i} - 88\hat{j})\text{m} \end{aligned}$$

- (c) Find the average velocity between $t = 1\text{s}$ and $t = 5\text{s}$. (1 P)

Solution:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-8\hat{i} - 88\hat{j})\text{m}}{5\text{s} - 1\text{s}} = (-2\hat{i} - 22\hat{j})\frac{\text{m}}{\text{s}}$$

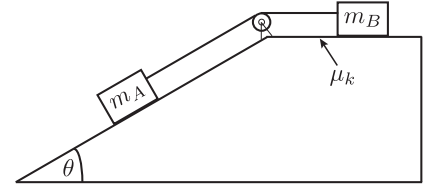
- (d) Find velocity vector and the acceleration vector at any time t . (2 P)

Solution:

$$\vec{v}(t) = -2\hat{i} - (6t + 4)\hat{j}, \quad \vec{a}(t) = -6\hat{j}$$

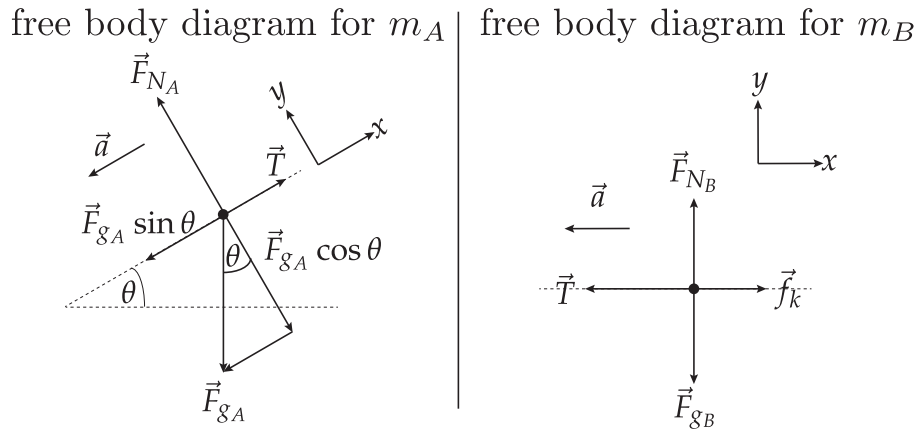
4. Two blocks $m_A = 4\text{kg}$ and $m_B = 2\text{kg}$ are connected over a massless pulley by a massless cord. The coefficient of kinetic friction between block B and the horizontal plane is $\mu_k = 0.5$. The **inclined plane is frictionless** and at angle $\theta = 30^\circ$.

- (a) Draw the free body diagrams for the masses m_A and m_B . (4P)
- (b) Calculate the tension in the cord and the magnitude of the acceleration of the blocks. (5P)



Solution:

- (a) Free body diagrams for m_A and m_B



- (b) From the free body diagrams we get the following equations

From the free body diagram for m_A we get:

$$\sum \vec{F} = (T - F_{gA} \sin \theta) \hat{i} + (F_{N_A} - F_{gA} \cos \theta) \hat{j} = -m_A a \hat{i}$$

$$T - F_{gA} \sin \theta = -m_A a \quad (1)$$

$$F_{N_A} - F_{gA} \cos \theta = 0 \quad (2)$$

From the free body diagram for m_B we get:

$$\sum \vec{F} = (f_k - T) \hat{i} + (F_{N_B} - F_{gB}) \hat{j} = -m_B a \hat{i}$$

$$f_k - T = -m_B a \quad (3)$$

$$F_{N_B} - F_{gB} = 0 \quad (4)$$

From equations (3) and (4) we get

$$T = m_B a + m_B g \mu_k \quad (5)$$

Substitute T from (5) into (2)

$$(m_B a + m_B g \mu_k) - m_A g \sin \theta = -m_A a \quad (6)$$

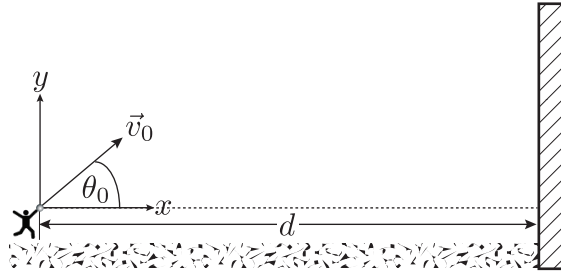
Solve (6) for a

$$a = \frac{m_A \sin \theta - m_B \mu_k}{m_A + m_B} g = \frac{4\text{kg} \sin 30^\circ - 2\text{kg} \times 0.5}{4\text{kg} + 2\text{kg}} 9.8 \frac{\text{m}}{\text{s}^2} = 1.63 \frac{\text{m}}{\text{s}^2}$$

Then we get for T :

$$T = m_B a + m_B g \mu_k = 2\text{kg} \times 1.63 \frac{\text{m}}{\text{s}^2} + 2\text{kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 0.5 = 13.06\text{N}$$

5. A person throws a ball toward a wall at an initial speed of 25m/s and an angle of $\theta_0 = 40^\circ$ above the horizontal. The wall is at a distance of $d = 22\text{m}$ from the release point of the ball.



- (a) At what point does the ball hit the wall, $(x, y) = ?$ (3P)

Solution:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 = (v_0 \cos \theta \hat{i} + v_0 \sin \theta \hat{j}) t + \frac{1}{2} (-g \hat{j}) t^2$$

With $x(T) = d$ we can calculate the time of flight until the ball hits the wall.

$$T = \frac{d}{v_0 \cos \theta} \quad (7)$$

Substitute T in $y(t)$.

$$\begin{aligned} y(T) &= v_0 \sin \theta \frac{d}{v_0 \cos \theta} - \frac{1}{2} g \left(\frac{d}{v_0 \cos \theta} \right)^2 = d \tan \theta - \frac{1}{2} g \left(\frac{d}{v_0 \cos \theta} \right)^2 = \\ &= 22\text{m} \tan 40^\circ - \frac{1}{2} 9.8 \frac{\text{m}}{\text{s}^2} \left(\frac{22\text{m}}{25 \frac{\text{m}}{\text{s}} \cos 40^\circ} \right)^2 = 11.99\text{m} \end{aligned}$$

So the point the ball hits the wall is $(x, y) = (22\text{m}, 11.99\text{m})$.

- (b) What are the x - and y -components of the velocity and the speed of the ball when it hits the wall? (3P)

Solution:

$$\begin{aligned} \vec{v}(t) &= \vec{v}_0 - \vec{g}t = v_0 \cos \theta \hat{i} + v_0 \sin \theta \hat{j} + (-g \hat{j}) t \\ \vec{v}(T) &= v_0 \cos \theta \hat{i} + \left(v_0 \sin \theta - g \frac{d}{v_0 \cos \theta} \right) \hat{j} \\ \vec{v}(T) &= 25 \frac{\text{m}}{\text{s}} \cos 40^\circ \hat{i} + \left(25 \frac{\text{m}}{\text{s}} \sin 40^\circ - 9.8 \frac{\text{m}}{\text{s}^2} \frac{d}{25 \frac{\text{m}}{\text{s}} \cos 40^\circ} \right) \hat{j} \\ \vec{v}(T) &= (19.15 \hat{i} + 4.81 \hat{j}) \frac{\text{m}}{\text{s}} \end{aligned}$$

The speed is then at the point of impact:

$$v(T) = |\vec{v}(T)| = \sqrt{\left(19.15 \frac{\text{m}}{\text{s}}\right)^2 + \left(4.81 \frac{\text{m}}{\text{s}}\right)^2} = 19.74 \frac{\text{m}}{\text{s}}$$

- (c) When it hits the wall has it passed the highest point of its trajectory or not? (3P)

Solution:

No, as the y -component of the velocity is positive, i.e. the direction of the ball is still upward, the highest point of the trajectory will be at a distance $> d$.