# PHYS101 Midterm Exam - Solution Set

Department of Physics

Spring 2014/15 - May 06, 2015



**Questions:**

1. Given $\vec{A} = (4\hat{i} + 6\hat{j})m$ and $\vec{B} = (-3\hat{i} + 7\hat{j})m$

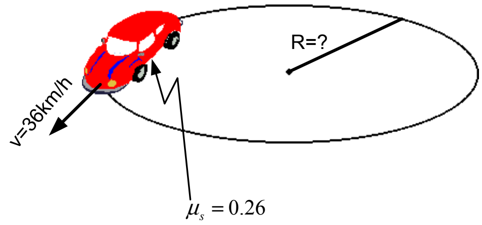
   (a) Find the vector $\vec{D} = \frac{1}{2}\vec{A} + 2\vec{B}$ in unit vector notation. (2 P)
   **Solution:**
   $$\vec{D} = \frac{1}{2}(4\hat{i} + 6\hat{j})m + 2(-3\hat{i} + 7\hat{j})m = (-4\hat{i} + 17\hat{j})m$$

   (b) Find the magnitude of the vector $\vec{D}$. (2 P)
   **Solution:**
   $$\left|\vec{D}\right| = \sqrt{(-4m)^2 + (17m)^2} = 17.46m$$

   (c) Find the angle the vector $\vec{D}$ makes with the positive $x$-axis. (2 P)
   **Solution:**
   $$\theta = \tan^{-1}\left(\frac{17}{-4}\right) = -76.76^\circ$$
   As the vector is the 2nd quadrant the angle becomes
   $$\theta = -76.76^\circ + 180^\circ = 103.24^\circ$$

   .

2. What is the smallest radius of an unbanked (flat) track around which a car can travel at a speed of $36km/h$ and the coefficient of static friction between the tires and the road is $\mu_s = 0.26$? (5P)




**Solution:**

$$f_s = mg\mu_s, F_c = m\frac{v^2}{r}$$

The centripetal force must be equal to the static frictional force

$$mg\mu_s = m\frac{v^2}{r} \Longleftrightarrow r = \frac{v^2}{\mu_s g} = \frac{(10m/s)^2}{0.26 \times 9.8\frac{m}{s^2}} = 39.24m$$

3. A particle moves in the $xy$-plane with the coordinates $x(t) = -2t + 3$ and $y(t) = -3t^2 - 4t$, with $x(t)$ and $y(t)$ in meters and $t$ in seconds.

   (a) Write the position vector. (1 P)
   **Solution:**
   $$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} = (-2t+3)\hat{\imath} + (-3t^2 - 4t)\hat{\jmath}$$

   (b) Find the displacement the particle between $t = 1s$ and $t = 5s$. (2 P)
   **Solution:**

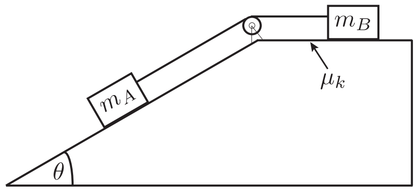
   $$\Delta\vec{r} = \vec{r}(5s) - \vec{r}(1s) = \Big[(-2\times 5 + 3)\hat{\imath} + (-3\times 5^2 - 4\times 5)\Big]\, m -$$
   $$- \Big[(-2\times 1 + 3)\hat{\imath} + (-3\times 1^2 - 4\times 1)\Big]\, m = \left(-8\hat{\imath} - 88\hat{\jmath}\right) m$$

   (c) Find the average velocity between $t = 1s$ and $t = 5s$. (1 P)
   **Solution:**

   $$\vec{v}_{avg} = \frac{\Delta r}{\Delta t} = \frac{\left(-8\hat{\imath} - 88\hat{\jmath}\right) m}{5s - 1s} = \left(-2\hat{\imath} - 22\hat{\jmath}\right)\frac{m}{s}$$

   (d) Find velocity vector and the acceleration vector at any time $t$. (2 P)
   **Solution:**
   $$\vec{v}(t) = -2\hat{\imath} - (6t+4)\hat{\jmath}, \qquad \vec{a}(t) = -6\hat{\jmath}$$

4. Two blocks $m_A = 4kg$ and $m_B = 2kg$ are connected over a massless pulley by a massless cord. The coefficient of kinetic friction between block $B$ and the horizontal plane is $\mu_k = 0.5$. The **inclined plane is frictionless** and at angle $\theta = 30°$.

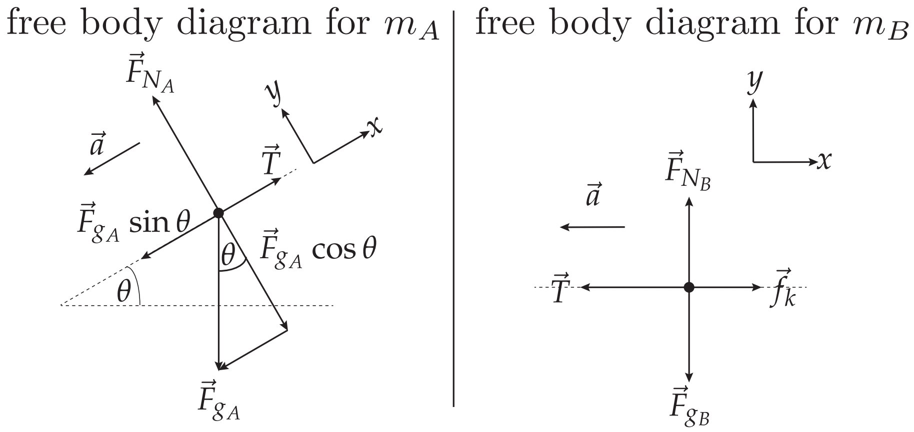



(a) Draw the free body diagrams for the masses $m_A$ and $m_B$. (4P)

(b) Calculate the tension in the cord and the magnitude of the acceleration of the blocks. (5P)

**Solution:**

(a) Free body diagrams for $m_A$ and $m_B$




(b) From the free body diagrams we get the following equations
From the free body diagram for $m_A$ we get:

$$\sum \vec{F} = (T - F_{g_A} \sin\theta)\,\hat{\imath} + (F_{N_A} - F_{g_A} \cos\theta)\,\hat{\jmath} = -m_A a \hat{\imath}$$

$$T - F_{g_A} \sin\theta = -m_A a \tag{1}$$

$$F_{N_A} - F_{g_A} \cos\theta = 0 \tag{2}$$

From the free body diagram for $m_B$ we get:

$$\sum \vec{F} = (f_k - T)\,\hat{\imath} + (F_{N_B} - F_{g_B})\,\hat{\jmath} = -m_B a \hat{\imath}$$

$$f_k - T = -m_B a \tag{3}$$

$$F_{N_B} - F_{g_B} = 0 \tag{4}$$

From equations (3) and (4) we get

$$T = m_B a + m_B g \mu_k \tag{5}$$

Substitute $T$ from (5) into (2)

$$(m_B a + m_B g \mu_k) - m_A g \sin\theta = -m_A a \tag{6}$$

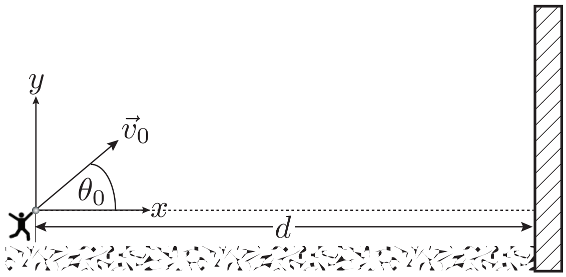
Solve (6) for $a$

$$a = \frac{m_A \sin\theta - m_B \mu_k}{m_A + m_B} g = \frac{4kg \sin 30° - 2kg \times 0.5}{4kg + 2kg} 9.8 \frac{m}{s^2} = 1.63 \frac{m}{s^2}$$

Then we get for $T$:

$$T = m_B a + m_B g \mu_k = 2kg \times 1.63 \frac{m}{s^2} + 2kg \times 9.8 \frac{m}{s^2} \times 0.5 = 13.06N$$

5. A person throws a ball toward a wall at an initial speed of $25m/s$ and an angle of $\theta_0 = 40°$ above the horizontal. The wall is at a distance of $d = 22m$ from the release point of the ball.




(a) At what point does the ball hit the wall, $(x, y) =$? (3P)
**Solution:**

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = (v_0 \cos\theta \hat{\imath} + v_0 \sin\theta \hat{\jmath})t + \frac{1}{2}(-g\hat{\jmath})t^2$$

With $x(T) = d$ we can calculate the time of flight until the ball hits the wall.

$$T = \frac{d}{v_0 \cos\theta} \tag{7}$$

Substitute $T$ in $y(t)$.

$$y(T) = v_0 \sin\theta \frac{d}{v_0 \cos\theta} - \frac{1}{2}g\left(\frac{d}{v_0\cos\theta}\right)^2 = d\tan\theta - \frac{1}{2}g\left(\frac{d}{v_0\cos\theta}\right)^2 =$$
$$= 22m\tan 40° - \frac{1}{2}9.8\frac{m}{s^2}\left(\frac{22m}{25\frac{m}{s}\cos 40°}\right)^2 = 11.99m$$

So the point the ball hits the wall is $(x, y) = (22m, 11.99m)$.

(b) What are the $x$- and $y$-components of the velocity and the speed of the ball when it hits the wall? (3P)
**Solution:**

$$\begin{aligned}
\vec{v}(t) &= \vec{v}_0 - \vec{g}t = v_0\cos\theta\hat{\imath} + v_0\sin\theta\hat{\jmath} + (-g\hat{\jmath})\,t \\
\vec{v}(T) &= v_0\cos\theta\hat{\imath} + \left(v_0\sin\theta - g\frac{d}{v_0\cos\theta}\right)\hat{\jmath} \\
\vec{v}(T) &= 25\frac{m}{s}\cos 40°\hat{\imath} + \left(25\frac{m}{s}\sin 40° - 9.8\frac{m}{s^2}\frac{d}{25\frac{m}{s}\cos 40°}\right)\hat{\jmath} \\
\vec{v}(T) &= (19.15\hat{\imath} + 4.81\hat{\jmath})\,\frac{m}{s}
\end{aligned}$$

The speed is then at the point of impact:

$$v(T) = |\vec{v}(T)| = \sqrt{\left(19.15\frac{m}{s}\right)^2 + \left(4.81\frac{m}{s}\right)^2} = 19.74\frac{m}{s}$$

(c) When it hits the wall has it passed the highest point of its trajectory or not?(3P)
**Solution:**
No, as the $y$-component of the velocity is positive, i.e. the direction of the ball is still upward, the highest point of the trajectory will be at a distance $> d$.