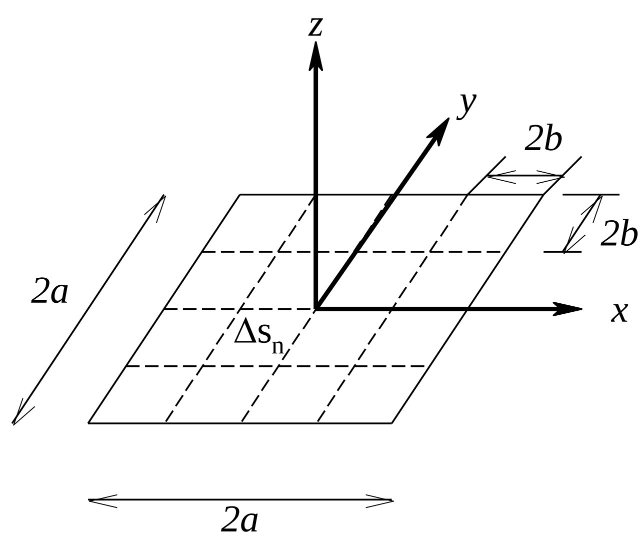
Moment Methods (Method of Moments

Charged Conducting Plate/ MoM Solution

- Consider a square conducting plate 2a meters on a side lying on the z=0 plane with center at the origin.



Let $\sigma(y,x)$ represent the surface charge density on the plate. Assume that the plate has zero thickness.

- Then, V(x,y,z):

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\int_{-a}^{a}\int_{-a}^{a}\frac{\sigma(x',y',z')}{R}dx'dy'$$

Where;

$$R = \left[(x - x')^2 + (y - y')^2 + z^2 \right]^{1/2}$$

When, $|x| < a, \quad |y| < a, \quad z = 0, \qquad V(x, y, z) \rightarrow V(const.)$

The Integral Equation:

$$4\pi\varepsilon_0 V = \int_{-a}^{a} dx' \int_{-a}^{a} dy' \frac{\sigma(x', y')}{\sqrt{(x - x')^2 + (y - y')^2}}$$

This is the integral equation for σ

Method of Moment Solution:

Consider that the plate is divided into N square subsections. Define:

$$f_n = \begin{cases} 1 & on\ \Delta S_n \\ 0\ on\ all\ other\ \Delta S_m \end{cases}$$

and let:

$$\sigma(x, y) \approx \sum_{n=1}^{N} \sigma_n f_n$$

Substituting this into the integral equation and satisfying the resultant equation at the midpoint (x_m, y_m) of each ΔS_m , we get:

$$V = \sum_{n=1}^{N} A_{mn}\sigma_n, \quad m = 1, 2, 3, \ldots, N$$

Where,

$$A_{mn} = \int_{\Delta x_n} dx' \int_{\Delta y_n} dy' \frac{1}{4\pi\varepsilon_0 \left[(x_m - x')^2 + (y_m - y')^2 \right]^{1/2}}$$

A_{mn} , is the potential at the center of ΔS_m due to a uniform charge density of unit amplitude over ΔS_n

Let :

- $2b = \frac{2a}{\sqrt{N}}$ denote the side length of each ΔS_n
- A_{nn} the potential at the center of due to the unit charge density over its own surface.

So,

$$A_{mn} = \int_{-b}^{b} dx \int_{-b}^{b} dy \frac{1}{4\pi\varepsilon_0 \sqrt{x^2 + y^2}}$$
$$= \frac{2b}{\pi\varepsilon_0} \ln(1+\sqrt{2})$$
$$= \frac{2b}{\pi\varepsilon_0}(0.8814)$$

- The potential at the center of ΔS_m can simply be evaluated by treating the charge over ΔS_n as if it were a point charge, so,

- So, the matrix equation:

$$A_{mn} \approx \frac{\Delta S_n}{4\pi\varepsilon_0 R_{mn}} = \frac{b^2}{\pi\varepsilon_0 \left[(x_m - x_n)^2 + (y_m - y_n)^2 \right]^{1/2}} \quad m \neq n$$

$$\left[A\right]\left[\alpha\right] = \left[V\right]$$
$$\left[\alpha\right] = \left[A\right]^{-1}\left[V\right]$$