Questions:

1. A flat puck of mass \( M = 2kg \) is rotating on a frictionless tabletop with radius \( R = 0.5m \). A light cord is connected to a block of mass \( m = 1kg \) through a central hole as shown in the figure.

(a) Find the magnitude of the tension in the cord. (2P)

Solution:

The reason for the tension in the cord is the weight of the dangling block, balanced with the centripetal force acting on the puck.

\[
T = mg = 1kg \cdot 9.8 \frac{m}{s^2} = 9.8N
\]

(b) Find the speed of the rotating puck. (5P)

Solution:

\[
T = \frac{M \pi^2}{R} \implies v = \sqrt{\frac{RT}{M}} = \sqrt{\frac{0.5m \cdot 9.8N}{2kg}} = 1.57 \frac{m}{s}
\]
Two blocks \( m_1 = 1.0 \text{kg} \) and \( m_2 = 4.0 \text{kg} \) are connected by a light string over a frictionless and massless pulley as shown in the figure. The coefficient of kinetic friction between the blocks and the surfaces is \( \mu_k = 0.25 \) and \( \theta = 30^\circ \).

(a) Draw the free body diagrams for \( m_1 \) and \( m_2 \). (5P)

**Solution:**

(a) [Free body diagrams for \( m_1 \) and \( m_2 \)]

(b) Calculate the magnitude of the acceleration of the blocks. (3P)

**Solution:**

\[
\begin{align*}
\sum \vec{F}_1 &= (T - f_{k_1}) \hat{i} + (F_{N_1} - m_1g) \hat{j} = m_1a \hat{i} \quad (1) \\
\sum \vec{F}_2 &= (-f_{k_2} - T + m_2g \sin \theta) \hat{i} + (F_{N_2} - m_2g \cos \theta) \hat{j} = m_2a \hat{i} \quad (2)
\end{align*}
\]

From equation (1) we get:

\[
T - f_{k_1} = m_1a \\ F_{N_1} - m_1g = 0 \implies F_{N_1} = m_1g \implies f_{k_1} = \mu_k m_1g \quad (3) \]

From equation (2) we get:

\[
-f_{k_2} - T + m_2g \sin \theta = m_2a \\ F_{N_2} - m_2g \cos \theta = 0 \implies F_{N_2} = m_2g \cos \theta \implies f_{k_2} = \mu_k m_2g \cos \theta \quad (4)
\]

So (3) and (5) become:

\[
\begin{align*}
T - \mu_k m_1g &= m_1a \\ -\mu_k m_2g \cos \theta - T + m_2g \sin \theta &= m_2a
\end{align*}
\]

(7)+(8) gives

\[
-\mu_k m_1g - \mu_k m_2g \cos \theta + m_2g \sin \theta = m_1a + m_2a
\]

Solving (9) for \( a \) gives

\[
a = \frac{-\mu_k (m_1 + m_2 \cos \theta) + m_2 \sin \theta}{m_1 + m_2} g = \\
= \frac{-0.25(1\text{kg} + 4\text{kg} \cos 30^\circ) + 4\text{kg} \sin 30^\circ}{1\text{kg} + 4\text{kg}} \times 9.8 \frac{m}{s^2} = 1.73 \frac{m}{s^2}
\]

(c) Calculate the magnitude of the tension in the string. (2P)

**Solution:**

Substituting (10) into (7) we get:

\[
T = \mu_k m_1g + m_1a = 0.25 \cdot 1\text{kg} \cdot 9.8 \frac{m}{s^2} + 1\text{kg} \cdot 1.73 \frac{m}{s^2} = 4.18 \text{N}
\]
3. A person pulls a 60 kg block from rest along a 50 m horizontal frictionless floor by a constant force $F_{app} = 120$N at an angle $\theta = 37^\circ$ with the horizontal, as shown in the figure below.

![Diagram](image)

(a) What is the work done by the applied force $F_{app}$ on the block? (2P)

**Solution:**
The work done by the applied force can be calculated straight forward

$$W_{app} = \vec{F}_{app} \cdot \Delta \vec{r} = (F_{app} \cos \theta \hat{i} + F_{app} \sin \theta \hat{j}) \cdot \Delta x \hat{i} = F_{app} \cos \theta \Delta x = 120 \text{N} \cos 37^\circ 50 \text{m} = 4792 \text{J}$$

(b) Using the Work-Kinetic Energy theorem, determine the speed of the block after it moves 50 m. (3P)

**Solution:**
$$\Delta K = W_{app}$$
$$K_f - K_i = W_{app}$$
$$\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = W_{app}$$
$$v_f = \sqrt{\frac{2}{m} \left( W_{app} + \frac{1}{2} mv_i^2 \right)} = \sqrt{\frac{2}{60 \text{kg}} (4792 \text{J} + 0)} = 12.6 \text{m/s}$$

(c) Assuming now the block experiences a rough surface with constant friction force $f_k = 50$N. What is the final speed of the block after being pulled for 50 m? (3P)

**Solution:**
The net force applied on the block is

$$\sum \vec{F} = (F_{app} \cos \theta - f_k) \hat{i} + (F_{app} \sin \theta + F_N - mg) \hat{j} = ma \hat{i}.$$ The work done by the net force is:

$$W = \sum \vec{F} \cdot \Delta \vec{r} = [(F_{app} \cos \theta - f_k) \hat{i} + (F_{app} \sin \theta + F_N - mg) \hat{j}] \cdot \Delta x \hat{i} = (F_{app} \cos \theta - f_k) \Delta x = (120 \text{N} \cos 37^\circ - 50 \text{N}) 50 \text{m} = 2292 \text{J}$$

Then we get for the final velocity:

$$v_f = \sqrt{\frac{2}{m} \left( W + \frac{1}{2} mv_i^2 \right)} = \sqrt{\frac{2}{60 \text{kg}} (2292 \text{J} + 0)} = 8.74 \text{m/s}$$
4. A block of mass $m = 10\text{kg}$ at rest slides down a rough incline plane of angle $\theta = 30^\circ$ and length $l = 5\text{m}$. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.1$. At the bottom of the plane the block continues to slide on a frictionless surface and hits a spring with spring constant $k = 100\text{N/m}$.

(a) Calculate the speed of the block at the point B of the incline. (5 P)

**Solution:**

The speed at the point B is the same as the speed at the bottom of the incline. There are two ways of solving this problem.

**Solution 1:** Block-incline-Earth system is isolated with non-conservative internal force acting.

The energy for the Block-Incline-Earth system is conserved so we can use:

$$\Delta E_{mech} = (K_f - K_i) + (U_{gf} - U_{gi}) = -f_k l$$

In order to calculate $f_k$ we need first to determine the normal force $F_N$, from the following free body diagram.

So we can read from the free body diagram:

$$F_N = mg \cos \theta,$$

and therefore:

$$f_k = \mu_k mg \cos \theta = 0.1 \cdot 10\text{kg} \cdot 9.8 \frac{m}{s^2} \cos 30^\circ = 8.49\text{N}.$$

$$\left(\frac{1}{2}mv_B^2 - 0\right) + (0 - mgh) = -f_k l$$

$$v_B = \sqrt{\frac{2}{m} (mgh - f_k l)}$$

with $h = l \sin \theta = 5\text{m} \sin 30^\circ = 2.5\text{m}$

we get for the speed at B:

$$v_B = \sqrt{\frac{2}{m} (mgh - f_k l)} = \sqrt{\frac{2}{10\text{kg}} \left(10\text{kg} \cdot 9.8 \frac{m}{s^2} \cdot 2.5\text{m} - 8.49\text{N} \cdot 5\text{m}\right)} = 6.36\frac{m}{s}$$

Alternatively, the second way of solving this problem is by considering the box alone system, then the gravitational force is an external force. The work done by the gravitational force is then:

$$W_{grav} = \mathbf{F}_g \cdot \Delta \mathbf{r} = (mg \sin \theta \mathbf{i} + mg \cos \theta \mathbf{j}) \cdot \mathbf{l} = mg \sin \theta l = 10\text{kg} \cdot 9.8 \frac{m}{s^2} \sin 30^\circ \cdot 5\text{m} = 245\text{J}$$
Then, the Work kinetic Energy theorem gives:

\[
\Delta K = W_{ext} - f_k l
\]
\[
\frac{1}{2}m v_B^2 - 0 = W_{ext} - f_k l
\]
\[
v_B = \sqrt{\frac{2}{m} (W_{ext} - f_k l)} = \sqrt{\frac{2}{m} (245 J - 8.49 N \times 5 m)} = 6.36 \frac{m}{s}
\]

(b) Calculate the maximum compression distance of the spring \(d\), the box needs to stop after hitting the spring. (5P)

**Solution:**

The Block-Surface-Spring system is isolated, with only conservative forces acting. So, the mechanical energy is conserved, i.e.

\[
\Delta K + \Delta U_g + \Delta U_{spring} = 0
\]
\[
\left(0 - \frac{1}{2}m v_B^2\right) + 0 + \left(\frac{1}{2} k d^2 - 0\right) = 0
\]

\[
d = \sqrt{\frac{m v_B^2}{k}} = \sqrt{\frac{10 kg \times (6.36 \frac{m}{s})^2}{100 N/m}} = 2.01 m
\]