Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

INFE221 – Electrical Circuits

Final Exam
Fall 2015-16

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Duration: 120 minutes

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Problem 1

Determine the output voltage $V_0$ in the circuit of Fig. P1

\[ \text{KCL at inverting input terminal yields:} \]
\[ \frac{5}{2k} + \frac{5-V_{01}}{8k} = 0 \]

Multiply both sides by $8k$

\[ 20 + 5 - V_{01} = 0 \Rightarrow V_{01} = 25 \text{ V} \]

\[ \text{KCL at } V_0:\]
\[ \frac{V_0 - 5}{6k} + \frac{V_0}{1k} + \frac{V_0 - V_{01}}{2k} = 0 \]

Multiply both sides by $6k$ and substitute the value of $V_{01}$:

\[ V_0 - 5 + 6V_0 + 3V_0 - 75 = 0 \]
\[ 10V_0 = 80 \]
\[ V_0 = 8 \text{ V} \]
**Problem 2**
Consider the circuit in Fig. P2. Under dc conditions, find:
(a) the power dissipated in 6Ω resistor and
(b) the energy stored in the capacitor and the inductors.

![Figure P2](image-url)

**Under dc conditions,** Inductors act like short circuits and capacitor acts like open circuit. Therefore the circuit becomes:

![KVL around loop I:](image-url)

\[-15 + v_c = 0\]

\[v_c = 15 \text{ V}\]

It is obvious that \[i_{L2} = -6 \text{ A}\]

Now, KVL around loop II:
\[-15 + 3i_{L1} + 6(i_{L1} - i_{L2}) = 0\]
\[9i_{L1} = 15 + 6i_{L2} = 15 + 6(-6) = -21\ A\]
\[\begin{align*}
i_{L1} &= \frac{-21}{9} = -\frac{7}{3}\ A
\end{align*}\]

The power dissipated in 6Ω resistor:
\[P_{6\Omega} = i^2 R = (i_{L1} - i_{L2})^2 6 = \left(-\frac{7}{3} - (-6)\right)^2 6\]
\[P_{6\Omega} = \left(\frac{11}{3}\right)^2 6 = \frac{121}{9} 6 = \frac{121}{3} 2 = \frac{242}{3} = 80.67\ W\]

The energy stored by 2 H inductor:
\[W_{2H} = \frac{1}{2} \left(\frac{-7}{3}\right)^2 = \frac{49}{9}\ J\]

The energy stored by 3 H inductor:
\[W_{3H} = \frac{1}{2} \left(3(-6)\right)^2 = 54J\]

The energy stored by 2F capacitor:
\[W_{2F} = \frac{1}{2} 2(15)^2 = 225J\]
Problem 3
Consider the circuit in Fig.P3. Assume the switch has been open for a long time and is closed at \( t = 0 \).

Find
a) \( v(0) \)
b) \( v(\infty) \)
c) Time constant \( \tau \).
d) \( v(t) \) for \( t > 0 \).

![Figure P3](image)

At \( t = 0^- \) (just before the switching action, the circuit is under dc conditions.)

It is obvious that \( v(0^-) \) is the voltage across \( 8k\Omega \) and \( 4k\Omega \) resistors. Therefore using voltage division principle:

\[
v(0^-) = 6 \times \frac{(8k + 4k)}{8k + 4k + 12k} = 3 \text{ V}
\]

Since the capacitor voltage cannot change instantaneously, \( v(0^+) = v(0^-) = v(0) \)
At $t = \infty$ the circuit is under dc conditions as well.

6kΩ and 12kΩ resistors are in parallel.

$6k \div 12k = \frac{6k \times 12k}{6k + 12k} = 4k$

Therefore

$v(\infty) = 6 \times \frac{8k + 4k}{8k + 4k + 4k} = 6 \times \frac{12}{16} = \frac{72}{16} = 4.5 \text{ V}$

Time constant $\tau$ is:

$\tau = R_{eq} C$

In order to find $R_{eq}$ all independent sources are set to zero.

$R_{eq} = 12k \div 6k \div (8k + 4k) = 3k$

$\tau = 3k \times 50 \mu s = 150 ms$

$v(t) = v(\infty) + \left[ v(0) - v(\infty) \right] e^{-\frac{t}{\tau}} = 4.5 + \left[ 3 - 4.5 \right] e^{-\frac{3}{150}} = 4.5 - 1.5 e^{-\frac{3}{150}} = 1.5 \left( 3 - e^{-\frac{3}{150}} \right)$
Problem 4
Determine \( i_x \) in the circuit of Fig. P4 using the phasor approach and superposition principle.

Using current division principle:

\[
I_x' = \frac{4[0 \left( -\frac{j}{4} \right)]}{1 - \frac{j}{4}} = \frac{4[0 \times \frac{1}{4} - 90]}{1.0308 \left| 14.036 \right|} = \frac{1 - 90}{1.0308 \left| 14.036 \right|} = 0.97 \left| -75.96 \right|
\]

\( i_x' = 0.96 \cos(4t - 75.96) \)

In order to find \( i_x'' \):

\[
I_x'' = -\frac{2[-90]}{1 - \frac{j}{2}} = -\frac{2[-90]}{1.118 \left| 26.565 \right|} = -1.79 \left| 63.435 \right| = 1.79 \left| 116.565 \right|
\]

\( i_x'' = 1.79 \cos(2t + 116.565) \)

\( i_x'' = 1.79 \sin(2t + 116.565 + 90) = 1.79 \sin(2t + 206.565) \)

Therefore
\[ i_x = i'_x + i''_x = 0.96 \cos(4t - 75.96) + 1.79 \sin(2t + 206.565) \]