

Tutorial 7: Linear Momentum and Collisions

Questions

1. A 3.00 kg particle has a velocity of $(3.00 \hat{i} - 4.00 \hat{j}) \frac{\text{m}}{\text{s}}$.
- (a) Find its x and y components of momentum.
- (b) Find the magnitude and direction of its momentum.

Solution:

Given Information

$$m = 3.00 \text{ kg}$$

$$\vec{v} = (3.00 \hat{i} - 4.00 \hat{j}) \frac{\text{m}}{\text{s}}$$

(a)

$$\vec{p} = m\vec{v} = 3.00 \text{ kg} [3.00 \hat{i} - 4.00 \hat{j}] \frac{\text{m}}{\text{s}} = 9.00 \hat{i} - 12.0 \hat{j} \text{ m/s.}$$

So we get for the x - and y -components:

$$p_x = 9.00 \text{ kg} \frac{\text{m}}{\text{s}}, \quad p_y = -12.0 \text{ kg} \frac{\text{m}}{\text{s}}$$

(b) The magnitude of the linear momentum yields to,

$$|\vec{p}| = \sqrt{p_x^2 + p_y^2} = \sqrt{9.00^2 + (-12.0)^2} \text{ kg} \frac{\text{m}}{\text{s}} = 15.0 \text{ kg} \frac{\text{m}}{\text{s}}$$

and the direction is given as

$$\tan \theta = \frac{p_y}{p_x} \implies \theta = \tan^{-1} \left(\frac{p_y}{p_x} \right) = \tan^{-1} \left(\frac{-12.0}{9.00} \right) = -59^\circ = 302^\circ.$$

2. A 65.0 kg boy and his 40.0 kg sister, both wearing roller blades, face each other at rest. The girl pushes the boy hard, sending him backward with velocity $2.90 \frac{\text{m}}{\text{s}}$ towards the west. Ignore friction.
- Describe the subsequent motion of the girl.
 - How much potential energy in the girl's body is converted into mechanical energy of the boy-girl system?
 - Is the momentum of the boy-girl system conserved in the pushing-apart process? If so, explain how that is possible considering
 - there are large forces acting and
 - there is no motion beforehand and plenty of motion afterward

Solution:

Let us first note down what information is given to us throughout the question.

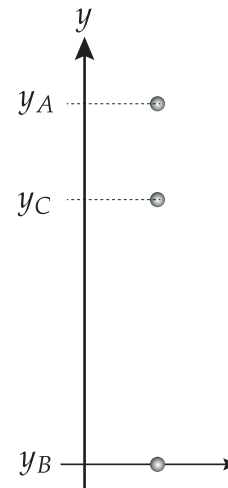
- $m_{boy} = 65.0 \text{ kg}, m_{girl} = 40.0 \text{ kg}$.
Ignore friction and take east to be positive.
 $Impulse = \vec{F}\Delta t = \Delta\vec{p}$
Having illustrated the equation for impulse, one can now evaluate impulse on the boy and the girl as follows.

$$\begin{aligned}\vec{I}_{boy} &= \Delta\vec{p}_{boy} = m_{boy} \left(-2.90 \hat{\mathbf{i}} \frac{\text{m}}{\text{s}} - 0 \right) \\ \vec{I}_{girl} &= \Delta\vec{p}_{girl} = m_{girl} (v_f \hat{\mathbf{i}} - 0)\end{aligned}$$

- $U_{girl} = TotalKineticEnergy$
- Yes, it is conserved.
- The system is isolated.
- Total momentum after = $65.0 \text{ kg} \left(-2.90 \frac{\text{m}}{\text{s}} \right) + 40.0 \text{ kg} \left(4.71 \frac{\text{m}}{\text{s}} \right) = 0$ [same as before].

3. After a 0.300 kg rubber ball is dropped from a height of $y_A = 1.75$ m, it bounces off a concrete floor ($y_B = 0$) and rebounds to a height of $y_C = 1.50$ m.

- (a) Determine the magnitude and direction of the impulse delivered to the ball by the floor.
- (b) Estimate the time the ball is in contact with the floor and use this estimate to calculate the average force the floor exerts on the ball.



Solution:

- (a)

$$\begin{aligned} \text{Impulse} &= \Delta \vec{p} & (1) \\ &= m(\vec{v}_f - \vec{v}_i) & (2) \end{aligned}$$

First we have to find the velocity of the rubber ball before it hits the ground. Therefore we will utilize the conservation of energy to calculate v_B . As we deal with an isolated system, the mechanical energy is conserved for the fall from y_A to y_B .

$$\begin{aligned} \Delta E_{\text{mech}} &= 0 \\ (K_f + U_f) - (K_i + U_i) &= 0 \\ \left(\frac{1}{2}mv_B^2 + 0\right) - (0 + mgy_A) &= 0 \\ \frac{1}{2}mv_B^2 &= mgy_A \\ v_B &= \sqrt{2gy_A} \\ &= \sqrt{2 \times 9.81 \frac{\text{m}}{\text{s}^2} \times 1.75 \text{ m}} \\ &= 5.86 \frac{\text{m}}{\text{s}} \end{aligned}$$

Now we have to calculate the velocity v'_B after it hits the ground inelastically. As the ball rises to 1.5 m above the ground the velocity v'_B can be calculated easily by the conservation of energy.

$$\begin{aligned}
 \Delta E_{mech} &= 0 \\
 (K_f + U_f) - (K_i + U_i) &= 0 \\
 \left(\frac{1}{2}mv_B'^2 + 0\right) - (0 + mgy_C) &= 0 \\
 \frac{1}{2}mv_B'^2 &= mgy_C \\
 v_B' &= \sqrt{2gy_C} \\
 v_B' &= \sqrt{2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 1.50 \text{ m}} \\
 &= 5.42 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \vec{I} &= \Delta \vec{p} \\
 &= (0.300 \text{ kg}) \left[5.42 \frac{\text{m}}{\text{s}} - \left(-5.86 \frac{\text{m}}{\text{s}} \right) \right] \hat{\mathbf{j}} = 3.38 \hat{\mathbf{j}} \text{ kg } \frac{\text{m}}{\text{s}}
 \end{aligned}$$

(b) Let $\Delta t = 0.05$ seconds. Since $\vec{I} = \vec{F}\Delta t$, then

$$\begin{aligned}
 \vec{I} &= \vec{F}\Delta t \\
 \vec{F} &= \frac{\vec{I}}{\Delta t} = \frac{3.38 \hat{\mathbf{j}} \text{ kg } \frac{\text{m}}{\text{s}}}{0.05 \text{ s}} = 7 \times 10^2 \hat{\mathbf{j}} \text{ N}
 \end{aligned}$$

4. In a slow-pitch softball game, a 0.200 kg softball crosses the plate at $15.0 \frac{\text{m}}{\text{s}}$ at an angle of 45.0° below the horizontal. The batter hits the ball toward centre field, giving it a velocity of $40.0 \frac{\text{m}}{\text{s}}$ at 30.0° above the horizontal.

(a) Determine the impulse delivered to the ball.

(b) If the force on the ball increases linearly for 4.00 ms, holds constant for 20.0 ms, and then decreases linearly to zero in another 4.00 ms, what is the maximum force on the ball?

Solution:

As batter is exerting an external force on the ball, therefore we just have to consider the change of the momentum of the ball to determine the impulse.

$$\begin{aligned}
 \vec{v}_i &= 15 \frac{\text{m}}{\text{s}} (\cos 45^\circ \hat{\mathbf{i}} - \sin 45^\circ \hat{\mathbf{j}}) = 10.6 \frac{\text{m}}{\text{s}} (\hat{\mathbf{i}} - \hat{\mathbf{j}}), \\
 \vec{v}_f &= 40 \frac{\text{m}}{\text{s}} (\cos 30^\circ \hat{\mathbf{i}} + \sin 30^\circ \hat{\mathbf{j}}) = (34.6 \hat{\mathbf{i}} + 20 \hat{\mathbf{j}}) \frac{\text{m}}{\text{s}},
 \end{aligned}$$

$$\begin{aligned}
 \vec{I} &= \Delta\vec{p}_{Ball} \\
 \Delta\vec{p}_{Ball} &= m(\vec{v}_f - \vec{v}_i) \\
 &= 0.2\text{ kg} \left((34.6\hat{i} - 20\hat{j}) \frac{\text{m}}{\text{s}} - 10.6 \frac{\text{m}}{\text{s}} (\hat{i} - \hat{j}) \right) \\
 &= (4.8\hat{i} - 1.9\hat{j}) \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Rearrange for v_{2f} .

$$\begin{aligned}
 \vec{v}_{2f} &= \frac{(5m_1 - 4.33\cos 30^\circ)\hat{i} - (4.33\sin 30^\circ)\hat{j}}{m_2} \text{ m/s} \\
 &= (1.25\hat{i} - 2.16\hat{j}) \text{ m/s}
 \end{aligned}$$

5. A railroad car of mass 2.50×10^4 kg is moving with a speed of 4.00 m/s. It collides and couples with three other coupled railroad cars, each of the same mass as the single car and moving in the same direction with an initial speed of 2.00 m/s.
 - (a) What is the speed of the four cars after the collision?
 - (b) How much mechanical energy is lost in the collision?

6. A 0.300-kg puck, initially at rest on a horizontal, frictionless surface, is struck by a 0.200-kg puck moving initially along the x axis with a speed of 2.00 m/s. After the collision, the 0.200-kg puck has a speed of 1.00 m/s at an angle of $\theta = 53.0^\circ$ to the positive x-axis.
 - (a) Determine the velocity of the 0.300kg puck after the collision.
 - (b) Find the fraction of kinetic energy transferred away or transformed to other forms of energy in the collision.

7. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.