PHYS102 Midterm Exam - Solution Set

Department of Physics

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Questions:

P1: Thermodynamics: (2+2+3 pts)

10 kg of ice at -20°C is dropped into a big lake at 0°C. (Assume that the lake is so big, so that temperature of the lake remains constant)

a) What is the equilibrium temperature of the system ice+lake?

Solution: $T_e = 0°C = 273K$

b) How much water of the lake is turned into ice?

Solution: The heat absorbed by ice to heat the ice from -20°C to 0°C is

$$Q_{ice} = m_{ice}c_{ice}\Delta T = 10kg \cdot 2100\frac{J}{kg \cdot K}\left(0-(-20)\right)K = 4.2 \times 10^5 J.$$

Q_{ice} is the heat extracted from the lake $\Longrightarrow Q_{lake} = -4.2 \times 10^5 J$. As the lake is at 0°C, this heat extraction leads to freezing a certain amount of ice, which can be calculated as following.

$$Q_{lake} = mL_f \Longrightarrow m = \frac{Q_{lake}}{L_f} = \frac{-4.2 \times 10^5 J}{-3.33 \times 10^5 \frac{J}{kg}} = 1.26kg$$

c) Calculate the change in entropy for the ice only, for the lake only, and for the isolated system (ice+lake). Does the result agree with the 2nd law of thermodynamics?

Solution:

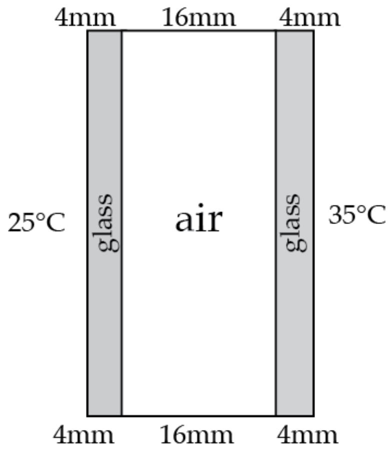
$$\begin{aligned}
\Delta S_{ice} &= \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{m_{ice}c_{ice}dT}{T} = m_{ice}c_{ice}\ln\left(\frac{T_f}{T_i}\right) = 10kg \cdot 2100\frac{J}{kg \cdot K}\ln\left(\frac{273K}{253K}\right) = 1598\frac{J}{K} \\
\Delta S_{lake} &= \int \frac{dQ}{T} = \frac{Q_{lake}}{T_{lake}} = \frac{-4.2 \times 10^5 J}{273K} = -1538\frac{J}{K}
\end{aligned}$$

The change of the entropy of the isolated system lake + ice is then:

$$\Delta S_{lake+ice} = \Delta S_{ice} + \Delta S_{lake} = 1598\frac{J}{K} + 1538\frac{J}{K} = 60\frac{J}{K} > 0$$

$\Delta S_{lake+ice} = 60\frac{J}{K} > 0$ agrees with the 2nd law of thermodynamics.

P2: Thermodynamics: (2+3+4 pts)
A typical balcony window in Cyprus has an area of $4m^2$, consists either of a single glass pane of thickness $4mm$, or a double glass window of two $4mm$ glass panes separated by a $16mm$ layer of air. On a regular summer day, the temperature outside is 35°C, and the air-condition keeps the temperature in the house at convenient 25°C.



a) Calculate the rate of heat flow through the single glass window.

Solution:

$$P_{cond_1} = Ak_{glass}\frac{T_H - T_C}{L} = 4m^2 \cdot 1\frac{W}{m \cdot K}\frac{(35-25)K}{4 \times 10^{-3}m} = 1.0 \times 10^4 W$$

b) Calculate the rate of heat flow through the double glass window with the air in between.

Solution:

$$P_{cond_2} = A\frac{T_H - T_C}{\sum_i \frac{L_i}{k_i}} A\frac{T_H - T_C}{\frac{L_{glass}}{k_{glass}} + \frac{L_{air}}{k_{air}} + \frac{L_{glass}}{k_{glass}}} = 4m^2 \cdot \frac{(35-25)K}{\frac{4\times10^{-3}m}{1\frac{W}{m\cdot K}} + \frac{16\times10^{-3}m}{0.026\frac{W}{m\cdot K}} + \frac{4\times10^{-3}m}{1\frac{W}{m\cdot K}}} = 64.2W$$

c) How much heat is lost in 2 hours through the single glass window and the three-layer window? Compare the results.

Solution:

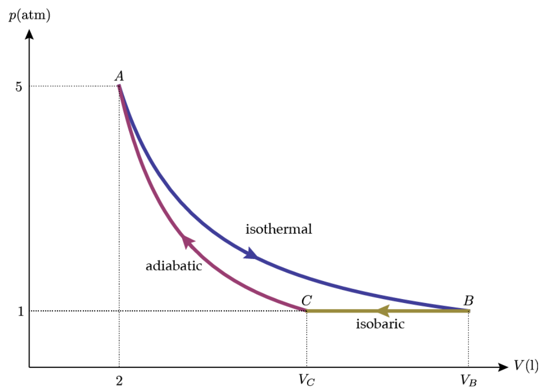
$$Q_1 = P_{cond_1} t = 1.0 \times 10^4 W \cdot 2h \cdot \frac{3600s}{1h} = 7.2 \times 10^7 J$$

$$Q_1 = P_{cond_2} t = 64.2W \cdot 2h \cdot \frac{3600s}{1h} = 4.62 \times 10^5 J$$

$$\frac{Q_1}{Q_2} = \frac{7.2 \times 10^7 J}{4.62 \times 10^5 J} = 156$$

$\Longrightarrow$ A single glass window looses 156 times more energy compared to a double glass window of $4m^2$ in the same time interval.

P3: Thermodynamics: (2+3+3+2 pts)
2 moles of a diatomic ideal gas undergo the following cyclic process, consisting of an isothermal, isobaric, and an adiabatic process. Given $p_A = 5atm$, $p_B = p_C = 1atm$, and $V_A 2lt$.



a) Calculate the Volumes V_B and V_C.

Solution:

Process $A \to B$ is isothermal $\Longrightarrow p_A V_A = nRT_A = nRT_B = p_B V_B \Longrightarrow V_B = \frac{p_A}{p_B}V_A = \frac{5atm}{1atm}2lt = 10lt$.

Process $C \to A$ adiabatic $\Longrightarrow p_A V_A^\gamma = p_C V_C^\gamma \Longrightarrow V_C = \left(\frac{p_A}{p_C}\right)^{\frac{1}{\gamma}} V_A = \left(\frac{5atm}{1atm}\right)^{\frac{1}{1.4}} 2lt = 6.3lt$

b) Calculate the work W_{AB}, W_{BC}, W_{CA}.

Solution:

Process $A \to B$ is isothermal, i.e. $T_A = T_B = const. \Longrightarrow p = \frac{nRT_A}{V} = \frac{p_A V_A}{V}$.

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} \frac{p_A V_A}{V}\,dV = p_A V_A \int_{V_A}^{V_B} \frac{dV}{V} = p_A V_A \ln\frac{V_B}{V_A} =$$

$$= 5atm \cdot 2lt \cdot 101\frac{J}{atm \cdot lt}\ln\frac{10lt}{2lt} = 1626J$$

Process $B \to C$ is isobaric, i.e. $p_B = p_C = const.$.

$$W_{BC} = \int_{V_B}^{V_C} p\,dV = \int_{V_B}^{V_C} p_B\,dV = p_B \int_{V_B}^{V_C} dV = p_B\,(V_C - V_B) = 1atm \cdot (6.3lt - 10lt) \cdot 101 \frac{J}{atm \cdot lt} = -373J$$

Process $C \to A$ is adiabatic.

Way 1:
Because $Q_{CA} = 0$,

$$W_{CA} = -\Delta E_{int_{CA}} = -nC_V \Delta T_C A = -\frac{5}{2}\,(nRT_A - nRT_C) = -\frac{5}{2}\,(p_A V_A - p_C V_C) =$$
$$= -\frac{5}{2}\,(5atm \cdot 2lt - 1atm \cdot 6.3lt) \cdot 101 \frac{J}{atm \cdot lt} = -932J$$

Way 2:
Because process $C \to A$ is adiabatic. $p_A V_A^\gamma = pV^\gamma \implies$
$p = \frac{p_A V_A^\gamma}{V^\gamma}$

$$W_{CA} = \int_{V_C}^{V_A} p\,dV = \int_{V_C}^{V_A} \frac{p_A V_A^\gamma}{V^\gamma}\,dV = \frac{p_A V_A^\gamma}{-\gamma+1}\left(V_A^{-\gamma+1} - V_C^{-\gamma+1}\right) =$$
$$= \frac{1}{-\gamma+1}\left(p_A V_A^\gamma V_A^{-\gamma+1} - \underbrace{p_A V_A^\gamma}_{=p_C V_C^\gamma} V_C^{-\gamma+1}\right) = \frac{1}{-\gamma+1}\,(p_A V_A - p_C V_C) =$$
$$= \frac{1}{-1.4+1}\,(5atm \cdot 2lt - 1atm \cdot 6.3lt) \cdot 101 \frac{J}{atm \cdot lt} = -932J$$

c) Calculate the Heat Q_{AB}, Q_{BC}, Q_{CA}.

Solution:

Process $A \to B$ is isothermal $\Delta E_{int_{AB}} = 0 = Q_{AB} - W_{AB} \implies$
$Q_{AB} = W_{AB} = 1626J$.

Process $B \to C$ is isobaric

$$Q_{BC} = nC_P \Delta T_{BC} = \frac{7}{2} nR(T_C - T_B) = \frac{7}{2}\,(p_C V_C - p_B V_B) =$$
$$= \frac{7}{2} p_B\,(V_C - V_B) = \frac{7}{2} 1atm\,(6.3lt - 10lt) \cdot 101 \frac{J}{atm \cdot lt} = -1308J.$$

Process $C \to A$ is adiabatic $\implies Q_{CA} = 0$

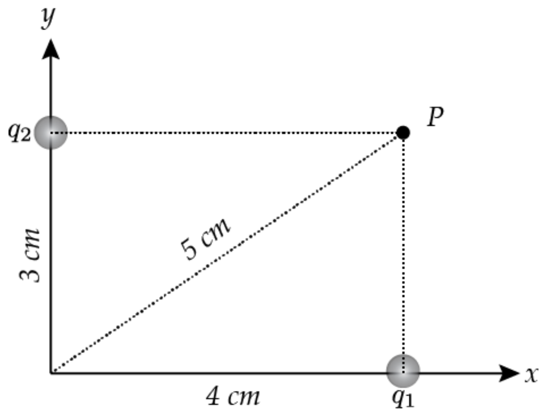
d) Calculate the thermal efficiency efficiency of this cycle.

Solution:

$$e = 1 - \frac{|Q_L|}{Q_H} = 1 - \frac{|Q_{BC}|}{Q_{AB}} = 1 - \frac{|-1308J|}{1626J} = 0.196 = 19.6\%$$

P4: Electricity: (2+2+1+2 pts)

Given the point charges $q_1 = 5\mu C$ located on the x-axis $4cm$from the origin and $q_2 = -2\mu C$ located on the y-axis $3cm$ from the origin.



a) Calculate the electric field generated by the charge q_1 at the position P, located at the position $\left(4\hat{\imath}+3\hat{\jmath}\right)\, cm$.

Solution:

$$\vec{E}_1 = k_e \frac{q_1}{r_1^2}\hat{r}_1 = 9\times 10^9 \frac{N}{m^2C^2}\frac{5\times 10^{-6}C}{(0.03m)^2}\hat{\jmath} = 5\times 10^7 \frac{N}{C}\hat{\jmath}$$

b) Calculate the electric field generated by the charge q_2 at the position P.

Solution:

$$\vec{E}_2 = k_e \frac{q_2}{r_2^2}\hat{r}_2 = 9\times 10^9 \frac{N}{m^2C^2}\frac{-2\times 10^{-6}C}{(0.04m)^2}\hat{\imath} = -1.125\times 10^7 \frac{N}{C}\hat{\imath}$$

c) Calculate the total electric field at the point P.

Solution:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 5\times 10^7 \frac{N}{C}\hat{\jmath} - 1.125\times 10^7 \frac{N}{C}\hat{\imath} = (-1.125\hat{\imath}+5\hat{\jmath})\times 10^7 \frac{N}{C}$$

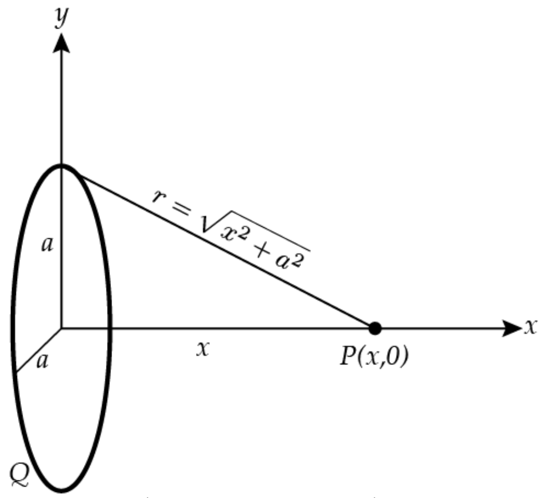
d) Calculate the force exerted by the charges q_1 and q_2 on a test charge $q_3 = -3\mu C$ located at P.

Solution:

$$\vec{F}_3 = q_3\vec{E} = -3\times 10^{-6}C(-1.125\hat{\imath}+5\hat{\jmath})\times 10^7\frac{N}{C} = (-33.75\hat{\imath}+150\hat{\jmath})\frac{N}{C}$$

P5: Electricity: (7 pts)

Calculate the electric field of a uniformly charged ring with a total charge Q and radius a , at a point P on the x-axis at a distance x from the centre of the ring.



Solution:

$$r = \sqrt{x^2 + a^2}, \quad \hat{r} = \frac{x\hat{i} - a\cos\theta\hat{j} - a\sin\theta\hat{k}}{\sqrt{x^2 + a^2}}$$

$$d\vec{E} = k\frac{dq}{r^2}\hat{r} = k\frac{\lambda a d\theta}{x^2 + a^2}\frac{x\hat{i} - a\cos\theta\hat{j} - a\sin\theta\hat{k}}{\sqrt{x^2 + a^2}}$$

integrating over θ gives:

$$\vec{E} = k\int_0^{2\pi}\left(\frac{\lambda a}{x^2 + a^2}\frac{x\hat{i} - a\cos\theta\hat{j} - a\sin\theta\hat{k}}{\sqrt{x^2 + a^2}}\right)d\theta =$$

$$= k\int_0^{2\pi}\left(\frac{\lambda a}{x^2 + a^2}\frac{x\hat{i}}{\sqrt{x^2 + a^2}}\right)d\theta + k\int_0^{2\pi}\left(\frac{\lambda a}{x^2 + a^2}\frac{-a\cos\theta\hat{j}}{\sqrt{x^2 + a^2}}\right)d\theta + k\int_0^{2\pi}\left(\frac{\lambda a}{x^2 + a^2}\frac{-a\sin\theta\hat{k}}{\sqrt{x^2 + a^2}}\right)d\theta =$$

$$= k\left(\frac{\lambda a x}{(x^2 + a^2)^{3/2}}\hat{i}\right)\int_0^{2\pi}d\theta - k\left(\frac{\lambda a^2}{(x^2 + a^2)^{3/2}}\hat{j}\right)\underbrace{\int_0^{2\pi}\cos\theta\, d\theta}_{=0} - k\left(\frac{\lambda a^2}{(x^2 + a^2)^{3/2}}\hat{j}\right)\underbrace{\int_0^{2\pi}\sin\theta\, d\theta}_{=0} =$$

$$= k\frac{\overbrace{\lambda a 2\pi}^{Q}\, x}{(x^2 + a^2)^{3/2}}\hat{i} = \frac{kQx}{(x^2 + a^2)^{3/2}}\hat{i}$$

of course you can argue directly that the component of the electric field in y and z direction cancel, as the calculation shows, and say

$$dE_x = k\frac{dq}{r^2} = k\frac{\lambda a d\theta}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

as there is no θ-dependence of x and a, the integral over θ gives 2π, resulting

$$E_x = k\frac{\overbrace{\lambda a 2\pi}^{Q}\, x}{(x^2 + a^2)^{3/2}} = \frac{kQx}{(x^2 + a^2)^{3/2}}.$$