**Question 1:** The energy absorbed by the BOX in Fig. P-1 is given below. Calculate and sketch the current flowing into the BOX. Also calculate the charge that enters the BOX between 0 and 12 seconds.

![Diagram of an electric circuit with a battery and a BOX](image)

**Solution**

In an electric circuit, the Power is the time rate of expending or absorbing energy, measured in watts (W)

\[ P = \frac{dw}{dt} \]

Whether the power supplied or absorbed by an element is defined as the product of the voltage across the element and the current through it.

\[ P = v i \]
<table>
<thead>
<tr>
<th>The domain ranges of the energy function measured in Second</th>
<th>The energy at each time interval measured in Joule</th>
<th>The power at each time interval measured in Watt</th>
<th>The current at each time interval measured in Ampere</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t1 = [0,2] )</td>
<td>( w = \frac{5}{2} t1 )</td>
<td>( p = 2.5 )</td>
<td>( i = 0.25 )</td>
</tr>
<tr>
<td>( t2 = [2,4] )</td>
<td>( w = 5 )</td>
<td>( p = 0 )</td>
<td>( i = 0 )</td>
</tr>
<tr>
<td>( t3 = [4,6] )</td>
<td>( w = -\frac{5}{2} t3 + 15 )</td>
<td>( p = -2.5 )</td>
<td>( i = -0.25 )</td>
</tr>
<tr>
<td>( t4 = [6,7] )</td>
<td>( w = 0 )</td>
<td>( p = 0 )</td>
<td>( i = 0 )</td>
</tr>
<tr>
<td>( t5 = [7,8] )</td>
<td>( w = -\frac{5}{2} t5 + \frac{35}{2} )</td>
<td>( p = -2.5 )</td>
<td>( i = -0.25 )</td>
</tr>
<tr>
<td>( t6 = [8,10] )</td>
<td>( w = -2.5 )</td>
<td>( p = 0 )</td>
<td>( i = 0 )</td>
</tr>
<tr>
<td>( t7 = [10,12] )</td>
<td>( w = \frac{5}{4} t7 - 15 )</td>
<td>( p = \frac{5}{4} )</td>
<td>( i = 0.125 )</td>
</tr>
</tbody>
</table>

\[
i(t) = \begin{cases} 
0.25; & 0 \leq t \leq 2 \\
0; & 2 \leq t \leq 4 \\
-0.25; & 4 \leq t \leq 6 \\
0; & 6 \leq t \leq 7 \\
-0.25; & 7 \leq t \leq 8 \\
0; & 8 \leq t \leq 10 \\
0.125; & 10 \leq t \leq 12 
\end{cases}
\]

We can find the charge \( q \) from the following equation

\[
q \triangleq \int_{t=0}^{t} i(t) \, dt
\]
Subsequently,

\[ q = \int_{0}^{2} 0.25 \, dt + \int_{4}^{6} -0.25 \, dt + \int_{7}^{8} -0.25 \, dt + \int_{10}^{12} 0.125 \, dt \]

\[ q = 0.5 - 0.5 - 0.25 + 0.25 = 0 \, C \]
**Question 2:** Find \( I_0 \) in the network in Fig. P-2 using Tellegen’s Theorem. (This theorem has been introduced in the year of 1952 by Dutch Electrical Engineer Bernard D.H. Tellegen. This is a very useful theorem in network analysis. According to Tellegen’s theorem, the summation of instantaneous powers for the n number of branches in an electrical network is zero.)

![Figure P-2](image)

**Solution**

Consider \( v_k \) is the instantaneous voltage across the \( k^{\text{th}} \) branch and \( i_k \) is the instantaneous current flowing through this branch. According to the Tellegen’s Theorem, the following expression satisfied in the network stated in figure P-2

\[
\sum_{k=1}^{n} v_k i_k = 0
\]

In the figure P-2, the number of branch is 8. Then,

\[
v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6 + v_7 i_7 + v_8 i_8 = 0
\]

By substituting the values of \( v_k \) and \( i_k \) as stated in figure,

\[
(8 \times 6) + (10 \times 4) + (6 \times I_o) + (16 \times 2) + (6 \times 1) + (8 \times 3) - (8 \times 3) - (24 \times 6) = 0
\]
Then,\
\[(6 \times I_o) = 18\]

\[I_o = 3 \text{ Amps}\]

**Question 3:** Find the equivalent resistance looking in at terminals a-b in the circuit in Fig.
Solution

Step 1
Step 4

\[ 4.8 + 7.2 \]

\[ 3.33 + 6.67 \]

\[ \frac{12}{10} \]

Step 5
Step 6

**Question 3:** If $V_2 = 4$ V in Fig. P-4, calculate $V_x$. 

$$V_x = 8 + 4 + 5.45$$

**Final Solution**
Solution We can notice clearly from the figure, the number of meshes fewer than the nodes and the number of voltage sources is higher than the number of current sources. Successively this circuit is more suitable for Mesh Analysis than Node analysis.

- The number of branches is equal to 11
- The number of nodes is equal to 8
- The number of independent loops $11 - 8 + 1 = 4$
- We can notice that the current source exists only in one mesh, consequently there is no super mesh in the circuit.
- The number of times that applying KVL is possible in the circuit is equal to the number of the independent loops minus the number of sources existing in the circuit, so we can apply KVL in the first and second and third mish.
The following equations express applying the KVL in the first second and third mesh subsequently

\[ 24 i_1 - 6 i_3 = -24 \]  \hspace{1cm} (1)

\[ 8 i_2 - 2 i_4 = 24 \]  \hspace{1cm} (2)

\[ -16 i_1 + 33 i_3 - 2 i_4 = V_x \]  \hspace{1cm} (3)

In the fourth mesh the following equation is satisfied

\[ i_4 = -2 I_1 \]  \hspace{1cm} (4)

Moreover,

\[ i_1 = - I_1 \]  \hspace{1cm} (5)

The following equation is derived from the assumption stated in the question text

\[ 2 i_2 = 4 \]  \hspace{1cm} (6)

The system of equations (1) to (6) has solution as in the following table

<table>
<thead>
<tr>
<th>( i_1 )</th>
<th>(-2\ A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_2 )</td>
<td>(2\ A)</td>
</tr>
<tr>
<td>( i_3 )</td>
<td>(-1.5\ A)</td>
</tr>
<tr>
<td>( i_4 )</td>
<td>(-4\ A)</td>
</tr>
<tr>
<td>( V_x )</td>
<td>(-9.5\ V)</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>(2\ A)</td>
</tr>
</tbody>
</table>