Midterm EXAMINATION

November 18, 2017

Duration : 100 minutes

Number of Questions: 4

Good Luck

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
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<td>1</td>
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<td>TOTAL</td>
<td>100</td>
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Read the following instructions carefully:
1. Calculators are not allowed.
2. Switch off mobile phones and do not borrow any stationery from your friends.
3. In your solutions, show all details you claim credit for.
Question 1

a) Convert the following number with the indicated base to decimal. (Show your steps and only the first two digits after the decimal point.) (5 pts.)

\[(245.34)_6 = 2 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 + 3 \times 6^{-1} + 4 \times 6^{-2}
\]

\[= 72 + 24 + 5 + \frac{3}{6} + \frac{4}{36} = 101 + \frac{22}{36} = 101.61\]

b) Convert the following decimal numbers to numbers in base 5. (Show your steps.) (6 pts.)

i. Integer Remainder

<table>
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<th>Remainder</th>
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<tr>
<td>48</td>
<td>9</td>
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<td>3</td>
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</table>

\[(48)_{10} = (143)_5\]

ii. 0.88

<table>
<thead>
<tr>
<th>Integer</th>
<th>Fraction</th>
<th>Coefficient</th>
</tr>
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<tbody>
<tr>
<td>0.88 x 5= 4</td>
<td>+ 0.4</td>
<td>a₁ = 4</td>
</tr>
<tr>
<td>0.4 x 5 = 2</td>
<td>+ 0.0</td>
<td>a₂ = 2</td>
</tr>
</tbody>
</table>

\[(0.88)_{10} = (0.42)_5\]
c) Convert \((1110111110.01101011001\) to hexadecimal and to octal. (4 pts.)

\[001110111110.01101000\]

\[= (3BE.68)_{16}\]

\[= (1676.32)_{8}\]

\[\left(\begin{array}{cccccc}
0 & 0 & 1 & 1 & 1 & 1 \\
1 & 6 & 7 & 6 & 3 & 2
\end{array}\right)_{2}\]

d) Convert \(+31\) and \(+56\) to binary, using the **signed-1’s complement** representation and **enough digit to accommodate** the numbers. Then perform the binary equivalent of \((-31) \text{ } + (+56)\) and \((-31) \text{ } - (+56)\). (10 pts.)

\[\begin{array}{c}
\text{('10' of) } 31 \\
\text{('10' of) } 56
\end{array}\]

\[\begin{array}{c}
\text{('01' of) } 31 \\
\text{('01' of) } 56
\end{array}\]

\[\begin{array}{c}
(31)_{io} = (11111)_{2} \\
(56)_{io} = (111000)_{2} \\
(31)_{io} + (56)_{io} = (87)_{io} = (1010111)_{2}
\end{array}\]

Therefore

\[(+87)_{io} = (01010111)_{2} \Rightarrow \text{requires 8 bits.}\]

\[(+31)_{io} = (00011111)_{2} \text{ signed-1’s complement of } (-31)_{io} = (11100000)\]

\[(+56)_{io} = (00111000)_{2} \text{ signed-1’s complement of } (-56)_{io} = (11000111)_{2}\]

\[\begin{array}{c}
\text{(-31)} \\
\text{(+56)}
\end{array}\]

\[\begin{array}{c}
\text{ (+2 5)}
\end{array}\]

\[\begin{array}{c}
\text{+0 0 1 1 0 0 0 0}
\end{array}\]

\[\text{+0 0 0 1 1 0 0 0 0}
\]

\[\begin{array}{c}
\text{+1}
\end{array}\]

\[\text{0 0 0 1 1 0 0 1}
\]

\[\text{(-31) - (+56) = (-31) + (-56)}
\]

\[\begin{array}{c}
\text{(-3 1)} \\
\text{(-5 6)} \\
\text{(-8 7)}
\end{array}\]

\[\begin{array}{c}
\text{+1 1 0 0 0 0 1 1 1}
\end{array}\]

\[\text{+1 0 1 0 1 1 1 1 1}
\]

\[\text{0 1 0 0 0 1 0 0 0 0}
\]

Since the most significant bit is 1 that is a negative number. In order to find the result - (1’s complement of the result) = -(01010111) = -87_{10}
Question 2

a) Simplify the following Boolean functions to a minimum number of literals using algebraic manipulation.

\[ F(x, y, z) = x' y' z + xy' + yz + xyz' \]

i. \[ F(x, y, z) = y'(x'z + x) + y(z + xz') = y'(x' + x)(z + x) + y(z + z')(z + x) \]

\[ F(x, y, z) = (y' + y)(x + z) = (x + z) \]

\[ F(w, x, y, z) = \sum(0, 1, 2, 4, 6, 8, 10) \]

\[ F(w, x, y, z) = w'x' y' z' + w'x' y' z + w' x' y' z + w' x y' z' + w' x y' z + w' x y z' + w' x y z + w x y' z ' \]

b) \[ F(w, x, y, z) = w' x' y' + w' x' z + w' x z' + w x' z' \]

\[ F(w, x, y, z) = w' x' y' + w' z' (x' + x) + (w + w') x' z' \]

\[ F(w, x, y, z) = w' x' y' + w' z' + x' z' \]

(15 pts.)
b) Express the following function as a sum of minterms and as a product of maxterms (10 pts.)

\[
F(w, x, y, z) = w'(x \oplus y) + w(y \odot z)
\]

\[
F(w, x, y, z) = w'(x'y + xy') + w(y'z' + yz) = w'x'y + w'xy' + wy'z' + wyz
\]

\[
F(w, x, y, z) = w'x'y(z' + z) + w'xy'(z' + z) + w(x' + x)y'z' + w(x' + x)y'z
\]

\[
F(w, x, y, z) = w'x'y + w'x'yz + w'xyz + w'xy'z + wx'y'z' + wxy'z + wx'yz + wxyz
\]

\[
F(w, x, y, z) = m_2 + m_3 + m_4 + m_8 + m_{12} + m_{11} + m_{15}
\]

\[
F(w, x, y, z) = \sum (2, 3, 4, 5, 8, 11, 12, 15)
\]

and Product of Maxterms:

\[
F(w, x, y, z) = \prod (0, 1, 6, 7, 9, 10, 13, 14)
\]
Question 3

For a given Boolean function

\[ F(A,B,C,D) = \sum (2, 4, 7, 13, 14) \]

\[ d(A,B,C,D) = \sum ((A \oplus B \oplus C \oplus D)') \]

a) Determine the sum of products (SOP).

b) Implement \( F \) with only NAND gates.

c) Determine the product of sums (POS).

d) Implement \( F \) with only NOR gates.

(25 pts.)

For a given Boolean function

\[ F(A,B,C,D) = \sum (2, 4, 7, 13, 14) \]

\[ d(A,B,C,D) = \sum ((A \oplus B \oplus C \oplus D)') \]

a. Determine the sum of products (SOP).

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b. Implement \( F \) with only NAND gates.

\[ F = B + A'C \]

\[ F = (B'.(A'C)')' \]

\[ A' \]

\[ C \]

\[ B' \]

\[ F \]

F = B + A'C

F = (B'.(A'C)')'

\[ \]

c. Determine the product of sums (POS).
d. Implement $F$ with only NOR gates.

\[
F = (A'+B) (B+C)
\]

\[
F = (A'+B)' + (B+C)'
\]

\[
F = (A'+B) (B+C)
\]

\[
F' = AB' + B'C'
\]
Question 4

Analyse the following circuit

\[ \text{F1 = A.B, F2 = (B \oplus C)'}, \]
\[ \text{F3 = (A.B)' . C' = (A' + B') C' = A'C' + B'C'} \]
\[ \text{F4 = (((B \oplus C)')' . (A'C' + B'C)')} \]
\[ \text{= ((B \oplus C)' . (A'C' + B'C)')} \]
\[ \text{= (B \oplus C)' . (A + C) . (B + C)} \]
\[ \text{= (BC' + B'C) (AB + AC + BC + C)} \]
\[ \text{= ABC' + ABB'C + ACB'C + ACB'C' + BCB'C + CBC' + CB'C + CB'C} \]
\[ \text{= ABC' + AB'C' + B'C} \]
\[ \text{= ABC' + B'C (A + 1)} \]
\[ \text{= ABC' + B'C} \]
\[ \text{F5 = (ABC' + B'C')} \oplus A.B \]
a. Use the truth table and determine the output \( F \) as sum of minterms and product of max terms.

<table>
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<th>B</th>
<th>C</th>
<th>A'</th>
<th>B'</th>
<th>C'</th>
<th>A.B</th>
<th>ABC'</th>
<th>B'C</th>
<th>F4</th>
<th>F5</th>
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\[
F(A,B,C,D) = \bigoplus (1, 5, 7)
\]
\[
F(A,B,C,D) = \bigwedge (0, 2, 3, 4, 6)
\]

b. Use Karnaugh Map to simplify the equations obtained in (a).

\[ F = B'C + AC \]
\[ F' = C + A'B \]
\[ F = C' (A+B') \]

c. Implement the circuits with minimum number of gates.