PHYS102 Quiz - Solution Set

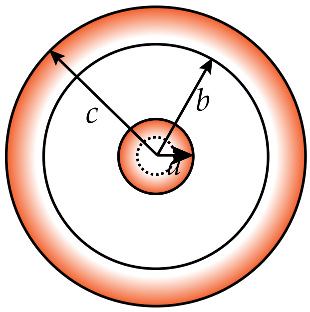
Department of Physics

Spring 2017 - May 16, 2017

Questions:
A solid conducting sphere of radius $a = 1cm$ has a charge $q_1 = 3\mu C$. Concentric with this sphere is a charged conducting shell with inner radius $b = 3cm$ and outer radius $c = 4cm$ having a charge $q_2 = -5\mu C$.

a) Calculate the electric field in the regions $r < a$.

Solution:

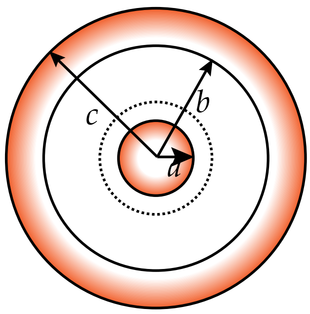


As the sphere is conducting all the charges are on the surface of the sphere, so the enclosed charge is 0. So, the electric field inside the conducting sphere is:

$$\vec{E} = 0 \text{ for } r < a$$

b) Calculate the electric field in the regions $a < r < b$.

Solution:
The Gaussian surface encloses the charge $q_1 = 3\mu C$. So, we get by application of Gausses Law



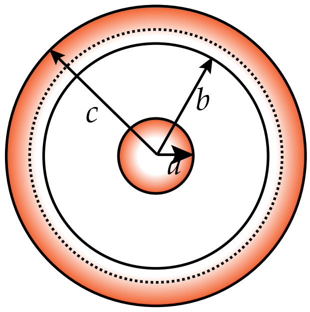
$$\oint \vec{E} \cdot d\vec{A} = \oint E \cdot dA = E \oint dA = E 4\pi r^2 = \frac{q_{encl}}{\varepsilon_0} = \frac{q_1}{\varepsilon_0}$$

Solving this equation for the electric field:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{r} = 9 \times 10^9 \frac{Nm^2}{C^2}\frac{3 \times 10^{-6}C}{r^2}\hat{r} = 2.7 \times 10^4 \frac{Nm^2}{C}\frac{\hat{r}}{r^2}$$

c) Calculate the electric field in the regions $b < r < c$.

Solution:



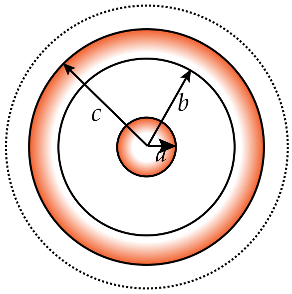
As the shell is conducting all the charges are on the surface of the conducting shell. The charge q_1 induces a charge $q_{ind1} = -q_1 = -3\mu C$ on the inner surface of the shell. So, the charge enclosed by the Gaussian surface is 0 and the the electric field inside the conducting shell becomes:

$$\vec{E} = 0 \text{ for } b < r < c$$

d) Calculate the electric field in the regions $r > c$.

Solution:

The Gaussian surface encloses the charge $q_1 + q_2 = 3\mu C - 5\mu C = -2\mu C$. So, we get by application of Gausses Law



$$\oint \vec{E} \cdot d\vec{A} = \oint E \cdot dA = E \oint dA = E 4\pi r^2 = \frac{q_{encl}}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0}$$

Solving this equation for the electric field:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 + q_2}{r^2} \hat{r} = 9 \times 10^9 \frac{Nm^2}{C^2} \frac{-2 \times 10^{-6} C}{r^2} \hat{r} = -1.8 \times 10^4 \frac{Nm^2}{C} \frac{\hat{r}}{r^2}$$

e) Calculate the potential difference $\Delta V_{ab} = V(b) - V(a)$.

Solution:

The electric field in the region from a to b is given according to question b) as:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r^2} \hat{r}$$

So, the electric potential can be determined as:

$$\Delta V_{ab} = -\int_a^b \vec{E} \cdot d\vec{\ell} = -\int_a^b \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r^2} \hat{r} \cdot \hat{r} d\vec{r} = -\frac{q_1}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} = k_e \left(\frac{q_1}{b} - \frac{q_1}{a} \right) =$$

$$= 9 \times 10^9 \frac{Nm^2}{C^2} \left(\frac{3 \times 10^{-6} C}{0.04 m} - \frac{3 \times 10^{-6} C}{0.03 m} \right) = -2.25 \times 10^5 V$$