Faculty of Engineering

ELECTRICAL AND ELECTRONIC ENGINEERING DEPARTMENT

EENG223 – Circuit Theory I

Final Exam
Spring 2014-15

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Duration: 120 minutes

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Problem 1

The mesh equations for the circuit shown in Fig.P1 are

\[
\begin{bmatrix}
8 & -2 & -5 \\
-2 & 5 & -3 \\
-5 & -3 & 18
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
=
\begin{bmatrix}
3 \\
3 \\
0
\end{bmatrix}
\]

Given \( V_3 = 5 \text{ V} \), find \( R_1, R_2, R_3, R_4, R_5 \), and \( V_1, V_2 \).
Problem 2
Using nodal analysis find $i_x$ in the circuit shown in Fig.P2.

![Figure P2](image)

KCL at V:

\[ \frac{V - 48}{16} + \frac{V - 96}{24} + \frac{V}{48} = i_x = \frac{V - 96}{24} \]

\[ \left( \frac{1}{16} + \frac{1}{48} \right) V = \frac{48}{16} = 3 \]

Multiply both sides of the equation by 48 yields:

\[ 4V = 3(48) \Rightarrow V = 36 \text{ V} \]

Therefore

\[ i_x = \frac{36 - 96}{24} = -\frac{60}{24} = -2.5 \text{ A} \]
Problem 3
Determine the current $i_0$ for the circuit shown in Fig.P3.

KCL at inverting terminal of OP AMP (1)

\[
\frac{7.8 - 5.8}{4k} + \frac{7.8 - V_{10}}{8k} = 0
\]

Multiply both sides by $8k$.

\[4 + 7.8 = V_{10} \Rightarrow V_{10} = 11.8 \text{ V}\]

KCL at inverting terminal of OP AMP (2)

\[
\frac{5.8 - 7.8}{4k} + \frac{5.8 - V_{20}}{8k} = 0
\]

Multiply both sides by $8k$.

\[-4 + 5.8 = V_{20} \Rightarrow V_{20} = 1.8 \text{ V}\]

\[i_0 = \frac{11.8 - 1.8}{4k} = 2.5 \text{ mA}\]
Problem 4
The circuit shown in Fig. P4 is under dc conditions before the switch closes at time $t = 0$. Determine $v(t)$ for $t > 0$.

![Circuit Diagram]

At $t = 0^-$
(since the circuit is under dc conditions, capacitor acts like open circuit)

![Circuit Diagram]

Using voltage division principle
$V(0^-) = 18 \frac{9}{27} = 6 \text{ V}$

Since the capacitor voltage cannot change instantaneously,
$V(0^-) = V(0^+) = 6 \text{ V}$

For $t > 0$

![Circuit Diagram]
Since the circuit contains a dc source:

\[ V(t) = V(\infty) + [V(0) - V(\infty)] e^{-\frac{t}{\tau}} \]

Where

\[ \tau = R_{eq} C \]

\( R_{eq} \) is the equivalent resistance seen by the capacitor by setting the independent source values to zero.

\[ R_{eq} = 9 \Omega / 9 = 4.5 \Omega \]

\[ \tau = 4.5 \times 250 \times 10^{-3} = \frac{4.5}{4} = 1.125 \text{ sec.} \]

At \( t = \infty \), the circuit will be under dc conditions again.

Using voltage division principle

\[ V(\infty) = 18 \times \frac{9}{18} = 9 \text{ V} \]

Therefore

\[ V(t) = 9 + [6 - 9] e^{-0.89t} \]

\[ V(t) = 9 - 3e^{-0.89t} \text{ V} \]
Problem 5
Suppose that the switch in Fig. P5 has been closed for a long time and is opened at $t = 0$. Find:

(a) $i(0^+)$ and $v(0^+)$

(b) $\frac{di(0^+)}{dt}$

(c) $i(t)$ for $t > 0$

Figure P5

(a) At $t = 0^-$, the circuit is under dc conditions and inductor acts like short circuit and capacitor acts like open circuit.

It is obvious that

$i(0^-) = 0$ and $v(0^-)$ is the voltage across $40\, \Omega$ resistor.

Therefore, using voltage division principle

$v(0^-) = 20 \frac{40}{40+10} = 16 \, V$

Since the capacitor voltage and inductor current cannot change instantaneously,
\( v(0+) = v(0-) = 16 \text{ V} \)
\( i(0+) = i(0-) = 0 \)

For \( t > 0 \)

(b) If we write KVL around the loop:

\[
2.5 \frac{di}{dt} + 40i + 60i + v = 0
\]
\[
2.5 \frac{di}{dt} + 100i + v = 0 \quad \text{.........(1)}
\]

and

\[ i = 1 \times 10^{-3} \frac{dv}{dt} \quad \text{.........(2)} \]

If we write Eqn.(1) at \( t = 0+ \)

\[
2.5 \frac{di(0+)}{dt} + 100i(0+) + v(0+) = 0
\]

\[
\frac{di(0+)}{dt} = -\frac{16}{2.5} = -6.4 \text{ A/s}
\]

(c) From Eqn.(1):

\[ v = -2.5 \frac{di}{dt} - 100i \quad \text{.........(3)} \]

Substitution of Eqn.(3) into (2) yields:

\[ i = -2.5 \times 10^{-3} \frac{d^2i}{dt^2} - 100 \times 10^{-3} \frac{di}{dt} \]

If we multiply both sides of the equation by 400 and rearrange it, we can obtain:

\[
\frac{d^2i}{dt^2} + 40 \frac{di}{dt} + 400i = 0
\]
Characteristic equation is
\[ s^2 + 40s + 400 = 0 \]
\[ (s + 20)(s + 20) = 0 \]

Which implies that
\[ s_{1,2} = -20 \text{ real and equal natural frequencies. (Critically damped case)} \]
\[ i(t) = e^{-20t} \left( A_1 + A_2t \right) \]
\[ i(0) = 0 = A_1 \]
\[ \frac{di}{dt} = -20e^{-20t} \left( A_1 + A_2t \right) + A_2e^{-20t} \]
\[ \frac{di(0)}{dt} = -6.4 = -20A_1 + A_2 \]
\[ A_2 = -6.4 \]
\[ i(t) = -6.4te^{-20t} A \]