Questions related to SECTION 2.2

1. Let

$$f(x) = \frac{x^3 - 1}{x - 1}$$

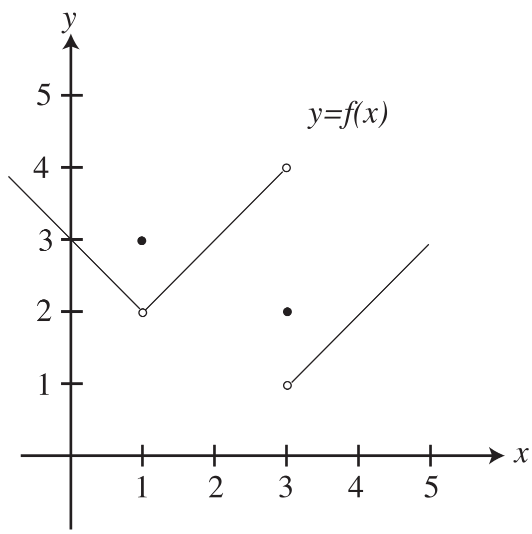
(a) Calculate $f(x)$ for each value of x in the following table.

x	0.9	0.99	0.999	0.9999	1	1.0001	1.001	1.01	1.1
f(x)	2.7	2.97	2.997	2.9997	undefined	3.0003	3.003	3.0301	3.31

(b) Make a conjecture about the value of

$$\lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 1}(x^2+x+1) = 1^2+1+1 = 3$$

2. Use the graph of f in the figure to find the following values, if they exists. If a limit does not exist, explain why.



(a) $f(1) = 3$

(b) $\lim_{x\to 1^-} f(x) = 2$

(c) $\lim_{x\to 1^+} f(x) = 2$

(d) $\lim_{x\to 1} f(x) = 2$

(e) $f(3) = 2$

(f) $\lim_{x \to 3^-} f(x) = 4$

(g) $\lim_{x \to 3^+} f(x) = 1$

(h) $\lim_{x \to 3} f(x)$ does not exist since $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

(i) $f(2) = 3$

(j) $\lim_{x \to 2^-} f(x) = 3$

(k) $\lim_{x \to 2^+} f(x) = 3$

(l) $\lim_{x \to 2} f(x) = 3$

Questions related to SECTION 2.3

1. Evaluate the following limits

(a) $\lim_{t \to -2} (t^2 + 5t + 7) = (-2)^2 + 5(-2) + 7 = 1$

(b) $\lim_{b \to 2} \dfrac{3b}{\sqrt{4b+1} - 1} = \dfrac{3(2)}{\sqrt{4(2)+1} - 1}$

2. Let $f(x) = \begin{cases} x^2 + 1, & \text{if } x < -1 \\ \sqrt{x+1}, & \text{if } x \geq -1 \end{cases}$

Compute the following limits or state that they do not exist.

(a) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x^2 + 1 = (-1)^2 + 1 = 2$

(b) $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \sqrt{x+1} = \sqrt{-1+1} = 0$

(c) $\lim_{x \to -1} f(x)$ does not exist since $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

3. Evaluate the following limits, where a and b are fixed real constants.

(a) $\lim_{x\to 4} \frac{x^2-16}{4-x} = \lim_{x\to 4} \frac{(x-4)(x+4)}{-(x-4)} = \lim_{x\to 4} -(x+4) = -(4+4) = -8$

(b) $\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h\to 0} \frac{\frac{5-5-h}{5(5+h)}}{h} = \lim_{h\to 0} \frac{-1}{5(5+h)} = -\frac{1}{5(5+0)} = -\frac{1}{25}$

(c)

$$\begin{aligned}
\lim_{t\to a} \frac{\sqrt{3t+1}-\sqrt{3a+1}}{t-a} &= \lim_{t\to a} \left(\frac{\sqrt{3t+1}-\sqrt{3a+1}}{t-a} \cdot \frac{\sqrt{3t+1}+\sqrt{3a+1}}{\sqrt{3t+1}+\sqrt{3a+1}} \right) \\
&= \lim_{t\to a} \frac{3t+1-3a-1}{(t-a)(\sqrt{3t+1}+\sqrt{3a+1})} \\
&= \lim_{t\to a} \frac{3(t-a)}{(t-a)(\sqrt{3t+1}+\sqrt{3a+1})} \\
&= \frac{3}{2\sqrt{3a+1}}
\end{aligned}$$

