MATH151 Quiz 1
Department of Mathematics, Spring 2014
March 17, 2014

Questions:

1. Find the limit

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=4$$

2. Find the limit

$$\lim_{x\to 7^-}\frac{1}{7-x}=\left|\begin{array}{c} x\to 7^=, x<7 \\ 7-x>0, 7-x\to 0^+ \end{array}\right|=\infty$$

3. Find the limit

$$\lim_{x\to\infty}\frac{3x^2+18x-35}{6x^2+4x-7}=$$

$$\left|\begin{array}{l} \text{degrees of the numerator and the denominator are equal} \\ \text{the limit is equal to the ratio of the leading coefficients} \end{array}\right|$$

$$=\frac{3}{6}=\frac{1}{2}.$$

4. Use the Squeeze theorem to show that

$$\lim_{x\to\infty}\frac{\sin x}{x^4}=0$$

$$\begin{aligned} -1 &< \sin x<1, \\ -\frac{1}{x^4} &< \frac{\sin x}{x^4}<\frac{1}{x^4}, \\ \lim_{x\to\infty}\frac{1}{x^4} &= 0, \lim_{x\to\infty}\left(-\frac{1}{x^4}\right)=0, \\ & \text{By the Squeeze Theorem,} \\ \lim_{x\to\infty}\frac{\sin x}{x^4} &= 0 \end{aligned}$$

5. Show that the function

$$f(x)=\begin{cases} \sqrt{x-1}, & x\geq 1 \\ x^2-1, & x<1 \end{cases}$$

is continuous at $x=1$.

$$\begin{aligned}
\lim_{x \to 1^{=}} f(x) &= \lim_{x \to 1^{=}} \left(x^2 - 1\right) = 0, \\
\lim_{x \to 1^{+}} f(x) &= \lim_{x \to 1^{=}} \sqrt{x - 1} = 0, \\
f(1) &= 0, \\
\lim_{x \to 1^{=}} f(x) &= \lim_{x \to 1^{+}} f(x) = f(1) = 0, \\
\text{the function is continuous at } x &= 1.
\end{aligned}$$