Guidelines for Curve Sketching

1. Find the domain of $f(x)$, i.e. all real numbers where $f(x)$ is defined.

2. Find $x$- and $y$- intercepts
   - $x$-intercept(s) are the solutions of the equation $f(x) = 0$, if any
   - $y$-intercept is the value of function $f(0)$, if it exists.

3. Find Critical Points and Local Extreme Values:
   - Find $f'(x)$.
   - Determine all critical points, i.e. the values of $x$ where $f'(x) = 0$ or $f'(x)$ is not defined.
   - Use the first derivative test to classify these local extreme values. Determine the intervals where the graph of the function is increasing, i.e. where $f'(x) > 0$, and where the graph of the function is decreasing, i.e. where $f'(x) < 0$.
   - Evaluate $f(x)$ at each critical point, wherever possible.

4. Determine the Concavity and Points of Inflection:
   - Find $f''(x)$.
   - Find all points where $f''(x) = 0$ or where $f''(x)$ is not defined.
   - Determine the intervals where the graph of $f(x)$ is concave up, i.e. $f''(x) > 0$, and where the graph of $f(x)$ is concave down, i.e. $f''(x) < 0$.
   - Determine whether the points found in 4b are points of inflection or not by checking if the sign of $f''(x)$ is changing at these points.

5. Find Asymptotes:
   - Determine the Limits $\lim_{x \to \infty} f(x) = L$, $\lim_{x \to -\infty} f(x) = M$, and $L$ and $M$ are finite, then $y = L$ or $y = M$ are horizontal asymptotes.
   - Check the left-hand and right-hand limits of the function at the points where $f(x)$ is not defined on the domain. If $\lim_{x \to a^-} f(x) = \pm \infty$ or $\lim_{x \to a^+} f(x) = \pm \infty$, then the line $x = a$ is a vertical asymptote. (In case of rational functions these points are the points where the denominator of the rational function becomes 0.)
   - Check if an oblique asymptote exists by determining the limit $\lim_{x \to \pm \infty} [f(x) - (ax + b)] = 0$.

6. Sketch the graph using the information gathered in 1-5

7. Determine the range of $f(x)$, i.e. all values $f(x)$ can take for all values of $x$ in the domain.