Guidelines for Curve Sketching

1. Find the domain of $f(x)$, i.e. all real numbers where $f(x)$ is defined.

2. Find x- and y- intercepts

   - x-intercept(s) are the solutions of the equation $f(x) = 0$, if any
   - y-intercept is the value of function $f(0)$, if it exists.

3. Find Critical Points and Local Extreme Values:

   (a) Find $f'(x)$.

   (b) Determine all critical points, i.e. the values of x where $f'(x) = 0$ or $f'(x)$ is not defined.

   (c) Use the first derivative test to classify these local extreme values. Determine the intervals where the graph of the function is **increasing** , i.e. where $f'(x) > 0$, and where the graph of the function is **decreasing**, i.e. where $f'(x) < 0$.

   (d) Evaluate $f(x)$ at each critical point, wherever possible.

4. Determine the Concavity and Points of Inflection:

   (a) Find $f''(x)$.

   (b) Find all points where $f''(x) = 0$ or where $f''(x)$ is not defined.

   (c) Determine the intervals where the graph of $f(x)$ is **concave up**, i.e. $f''(x) > 0$, and where the graph of $f(x)$ is **concave down**,i.e. $f''(x) < 0$.

   (d) Determine whether the points found in 4b are **points of inflection** or not by checking if the sign of $f''(x)$ is changing at these points.

5. Find Asymptotes:

   (a) Determine the Limits $\lim_{x\to\infty} f(x) = L$, $\lim_{x\to-\infty} f(x) = M$, and L and M are finite, then $y = L$ or $y = M$ are **horizontal asymptotes**.

   (b) Check the left-hand and right-hand limits of the function at the points where $f(x)$ is not defined on the domain. If $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$, then the line $x = a$ is a **vertical asymptote**. (In case of rational functions these points are the points where the denominator of the rational function becomes 0.)

   (c) Check if an **oblique asymptote** exists by determining the limit $\lim_{x\to\pm\infty} [f(x) - (ax + b)] = 0$.

6. Sketch the graph using the information gathered in 1-5

7. Determine the range of $f(x)$, i.e. all values $f(x)$ can take for all values of x in the domain.