

MATH151 Quiz 2  
Department of Mathematics, Spring 2014  
March 31, 2014

Questions:

1. Find the derivative of the following function

$$\begin{aligned}f(x) &= \frac{x^5 - 3x + 7}{x^3 - 4}, \\f'(x) &= \frac{(5x^4 - 3)(x^3 - 4) - 3x^2(x^5 - 3x + 7)}{(x^3 - 4)^2} \\&= \frac{5x^7 - 3x^3 - 20x^4 + 12 - 3x^7 + 9x^3 - 21x^2}{(x^3 - 4)^2} \\&= \frac{2x^7 - 20x^4 + 6x^3 - 21x^2 + 12}{(x^3 - 4)^2}\end{aligned}$$

2. Find the limit

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\tan 5x}{7x \cos(2x)} &= \lim_{x \rightarrow 0} \frac{5 \sin 5x}{7(5x) \cos(2x) \cos(5x)} = \\&= \lim_{x \rightarrow 0} \frac{5}{7 \cos(2x) \cos(5x)} \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = \frac{5}{7}.\end{aligned}$$

3. Find the derivative of the following function

$$\begin{aligned}f(x) &= [\sin(x^2) + 5x]^{125}, \\f'(x) &= 125 [\sin(x^2) + 5x]^{124} (\cos(x^2) 2x + 5)\end{aligned}$$

4. Find the derivative of the following function. Reduce your answer to a ratio

$$\begin{aligned}f(x) &= \sqrt[3]{x^2} (1 - x) = x^{2/3} (1 - x), \\f'(x) &= \frac{2}{3} x^{-1/3} (1 - x) - x^{2/3} = \frac{2(1 - x) - 3x}{3\sqrt[3]{x}} \\&= \frac{2 - 5x}{3\sqrt[3]{x}}\end{aligned}$$

5. Find the equation of the tangent line to the graph of the function given implicitly

$$x^4 - x^2y + y^4 = 1$$

at  $(-1, 1)$ .

$$\begin{aligned}4x^3 - 2xy - x^2y' + 4y^3y' &= 0, \\4x^3 - 2xy &= x^2y' - 4y^3y', \\y' &= \frac{4x^3 - 2xy}{x^2 - 4y^3}, \\y'((-1, 1)) &= \frac{-4 + 2}{1 - 4} = \frac{2}{3}, \\y - 1 &= \frac{2}{3}(x + 1).\end{aligned}$$