

MATH151 Quiz 3
Department of Mathematics, Spring 2014
April 12, 2014

Questions:

1. Use linear approximation of the function $f(x) = \sqrt[3]{x}$ to estimate $\sqrt[3]{28}$.
Choose a value of a to produce a small error.

$$\begin{aligned} a &= 27, & f'(x) &= \frac{1}{3\sqrt[3]{x^2}}, & f'(27) &= \frac{1}{27}, \\ f(27) &= \sqrt[3]{27} = 3, & \sqrt[3]{28} &\simeq 3 + \frac{1}{27}(28 - 27) = 3 + \frac{1}{27} = \\ &= \frac{82}{27} \end{aligned}$$

2. Find the limit using l'Hopital's Rule

$$\begin{aligned} \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x &= L \\ \ln L &= \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{3}{x}\right) \\ &= \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{3}{x}\right)}{1/x} = \left[\frac{0}{0}\right] \\ &= \lim_{x \rightarrow \infty} \frac{\frac{x}{x+3}(-3/x^2)}{-1/x^2} = 3 \\ L &= e^3. \end{aligned}$$

3. Use Part 1 of the Fundamental Theorem of Calculus to evaluate

$$\frac{d}{dx} \left[\int_1^x \sqrt{e^{2t} + 1} dt \right] = \sqrt{e^{2x} + 1}$$

4. Use Part 2 of the Fundamental Theorem of Calculus to evaluate

$$\begin{aligned} \int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx &= \left[\frac{2}{3} \sqrt{x^3} + 2\sqrt{x} \right]_4^9 = \left(\frac{54}{3} + 6 \right) - \left(\frac{16}{3} + 4 \right) \\ &= \frac{38}{3} + 2 = \frac{44}{3} \end{aligned}$$

5. Use the Substitution Rule to calculate the integral

$$\begin{aligned}\int x(x^2 + 3)^6 dx &= \left| \begin{array}{l} u = x^2 + 3 \\ du = 2x dx \end{array} \right| = \frac{1}{2} \int u^6 du \\ &= \frac{1}{2} u^7 + C = \frac{1}{2} (x^2 + 3)^7 + C\end{aligned}$$