

MATH151 Quiz 4
Department of Mathematics, Spring 2014
May 26, 2014 Duration 30 min

Questions:

1. Evaluate using integration by parts

$$\begin{aligned}\int x \cos x dx &= \left| \begin{array}{l} u = x, dv = \cos x dx \\ du = dx, v = \sin x \end{array} \right| = x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C\end{aligned}$$

2. Evaluate using partial fractions

$$\begin{aligned}\int \frac{dx}{x(x+5)} &= \int \left[\frac{A}{x} + \frac{B}{x+5} \right] dx, \\ \frac{1}{x(x+5)} &= \frac{A}{x} + \frac{B}{x+5} = \frac{A(x+5) + Bx}{x(x+5)} \\ &= \frac{(A+B)x + 5A}{x(x+5)}, \\ (A+B)x + 5A &= 1, \quad A = \frac{1}{5}, \quad A+B = 0, \quad B = -\frac{1}{5}. \\ \int \frac{dx}{x(x+5)} &= \int \left[\frac{1}{5x} - \frac{1}{5(x+5)} \right] dx = \frac{1}{5} (\ln|x| - \ln|x+5|) + C\end{aligned}$$

3. Evaluate the trigonometric integral

$$\begin{aligned}\int \cos^{10} x \sin^3 x dx &= \left| \begin{array}{l} \cos x = u \\ du = -\sin x dx \\ \sin^2 x = 1 - \cos^2 x = 1 - u^2 \end{array} \right| = - \int u^{10} (1 - u^2) du \\ &= \int (u^{12} - u^{10}) du = \frac{1}{13} u^{13} - \frac{1}{11} u^{11} + C \\ &= \frac{1}{13} \cos^{13} x - \frac{1}{11} \cos^{11} x + C\end{aligned}$$

4. Evaluate using the trigonometric substitution

$$\begin{aligned}\int \sqrt{4-x^2} dx &= \left| \begin{array}{l} x = 2 \sin t \\ dx = 2 \cos t dt \end{array} \right| = \int \sqrt{4-4\sin^2 t} 2 \cos t dt = \int \sqrt{4\cos^2 t} 2 \cos t dt \\ &= \int 4 \cos^2 t dt = 2 \int (1 + \cos 2t) dt = 2t + \sin 2t + C = 2t + \sin 2 \left(\sin^{-1} \left(\frac{x}{2} \right) \right) + C\end{aligned}$$

5. Evaluate the improper integral or state that it diverges

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{R \rightarrow \infty} \int_1^R \frac{1}{\sqrt{x}} dx = \lim_{R \rightarrow \infty} (2\sqrt{x}) \Big|_1^R = \lim_{R \rightarrow \infty} (2\sqrt{R} - 2) = \infty$$

The integral is divergent.