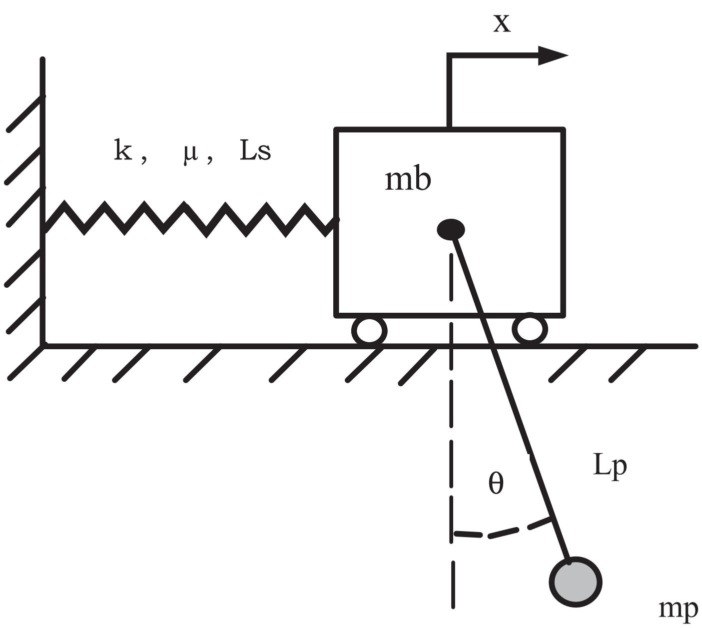

Figure D-1.

The variables are

mb	is the block mass,
k	is the spring stiffness,
μ	is mass density (mass/length) of the spring,
Ls	is the length of the spring,
mp	is the pendulum mass,
Lp	is the pendulum length,
x	is the absolute displacement of the mass,
θ	is the pendulum angular displacement.

The total potential energy is

$$PE = \frac{1}{2} k x^2 + mp\ g\ Lp\ (1 - \cos\theta) \tag{D-1}$$

The total kinetic energy is

$$KE = \frac{1}{2} mb\ \dot{x}^2 + \frac{1}{6} \mu\ \dot{x}^2\ Ls + \frac{1}{2} mp\ (Lp\dot{\theta} + \dot{x})^2 \tag{D-2}$$

Apply the energy method.

$$\frac{d}{dt}\left\{\frac{1}{2}mb\ \dot{x}^2 + \frac{1}{6}\mu\,\dot{x}^2\ Ls + \frac{1}{2}mp\ (Lp\dot{\theta}+\dot{x})^2 + \frac{1}{2}kx^2 + mp\ g\ Lp\ (1-\cos\theta)\right\} = 0 \tag{D-3}$$

$$mb\ \dot{x}\ddot{x} + \frac{1}{3}\mu\ \dot{x}\ddot{x}\ Ls + mp\ (Lp\dot{\theta}+\dot{x})(Lp\ddot{\theta}+\ddot{x}) + kx\ \dot{x} + mp\ g\ Lp\ (\sin\theta)\dot{\theta} = 0 \tag{D-4}$$

$$mb\ \dot{x}\ddot{x} + \frac{1}{3}\mu\ \dot{x}\ddot{x}\ Ls + mp\,Lp^2\,\dot{\theta}\ddot{\theta} + mp\,Lp\,\dot{\theta}\ddot{x} + mp\,Lp\ \ddot{\theta}\dot{x} + mp\ \dot{x}\ddot{x} \\ + kx\ \dot{x} + mp\ g\ Lp\ (\sin\theta)\,\dot{\theta} = 0 \tag{D-5}$$

For small angles,

$$\sin\theta \approx \theta \tag{D-6}$$

Thus,

$$mb\ \dot{x}\ddot{x} + \frac{1}{3}\mu\ \dot{x}\ddot{x}\ Ls + mp\,Lp^2\,\dot{\theta}\ddot{\theta} + mp\,Lp\,\dot{\theta}\ddot{x} + mp\,Lp\ \ddot{\theta}\dot{x} + mp\ \dot{x}\ddot{x} \\ + kx\ \dot{x} + mp\ g\ Lp\ \theta\ \dot{\theta} = 0 \tag{D-7}$$

$$+\left\{mb\ \ddot{x} + \frac{1}{3}\mu\ \ddot{x}\ Ls + mp\,Lp\ \ddot{\theta} + mp\ \ddot{x} + kx\right\}\dot{x} \\ + \left\{mp\,Lp^2\ \ddot{\theta} + mp\,Lp\ \ddot{x} + mp\ g\ Lp\ \theta\right\}\dot{\theta} = 0 \tag{D-8}$$

Equation (D-8) can be separated into two equations

$$\left\{ mb\ \ddot{x} + \frac{1}{3}\mu\ \ddot{x}\ Ls + mp\,Lp\ \ddot{\theta} + \ \ mp\ \ddot{x} + kx \right\}\dot{x} = 0$$

(D-9)

And

$$\left\{ mp\,Lp^2\ \ddot{\theta}\ + mp\,Lp\ \ddot{x} + mp\ g\ Lp\ \theta \right\}\dot{\theta} = 0$$

(D-10)

Divide each equation by its respect velocity term.

$$\left\{ mb\ \ddot{x} + \frac{1}{3}\mu\ \ddot{x}\ Ls + mp\,Lp\ \ddot{\theta} + \ \ mp\ \ddot{x} + kx \right\} = 0 \qquad \text{(D-11a)}$$

$$\left\{ \left[\ mb + mp + \frac{1}{3}\mu\ \ Ls \right] \ddot{x} + \ + mp\,Lp\ \ddot{\theta}\ \ + kx \right\} = 0 \qquad \text{(D-11b)}$$

And

$$\left\{ mp\,Lp^2\ \ddot{\theta}\ + mp\,Lp\ \ddot{x} + mp\ g\ Lp\ \theta \right\} = 0 \qquad \text{(D-12a)}$$

$$\left\{ mp\,Lp\ \ddot{\theta}\ + mp\ \ \ddot{x} + mp\ g\ \ \theta \right\} = 0 \qquad \text{(D-12b)}$$

$$\left\{ Lp\ \ddot{\theta}\ + \ \ddot{x} + g\ \ \theta \right\} = 0 \qquad \text{(D-12c)}$$

Assemble the equations in matrix form.

$$\begin{bmatrix} \left[\ mb + mp + \frac{1}{3}\mu\ \ Ls \right] & mp \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta}\,Lp \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(D-13)

Alternate Form

Again, the energy method can be used to derive the equation of motion. The following form can be used if the natural frequency is the only parameter of interest.

$$KE_1 + PE_1 = KE_2 + PE_2 \quad \text{(E-1)}$$

The subscripts represent time.

Conclusion

The energy method is suitable for reasonably simple systems.

The energy method may be inappropriate for complex systems, however. The reason is that the distribution of the vibration amplitude is required before the kinetic energy equation can be derived. Prior knowledge of the "mode shapes" is thus required.

The Lagrange method is better suited for complex systems, as discussed before.