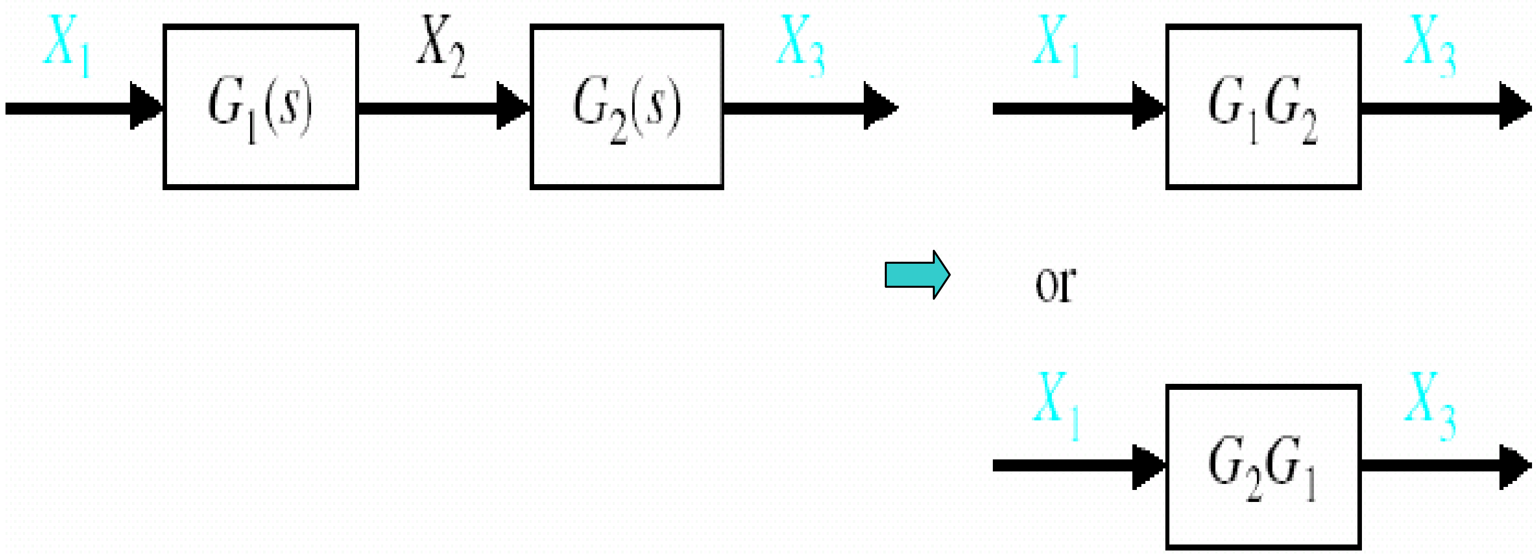
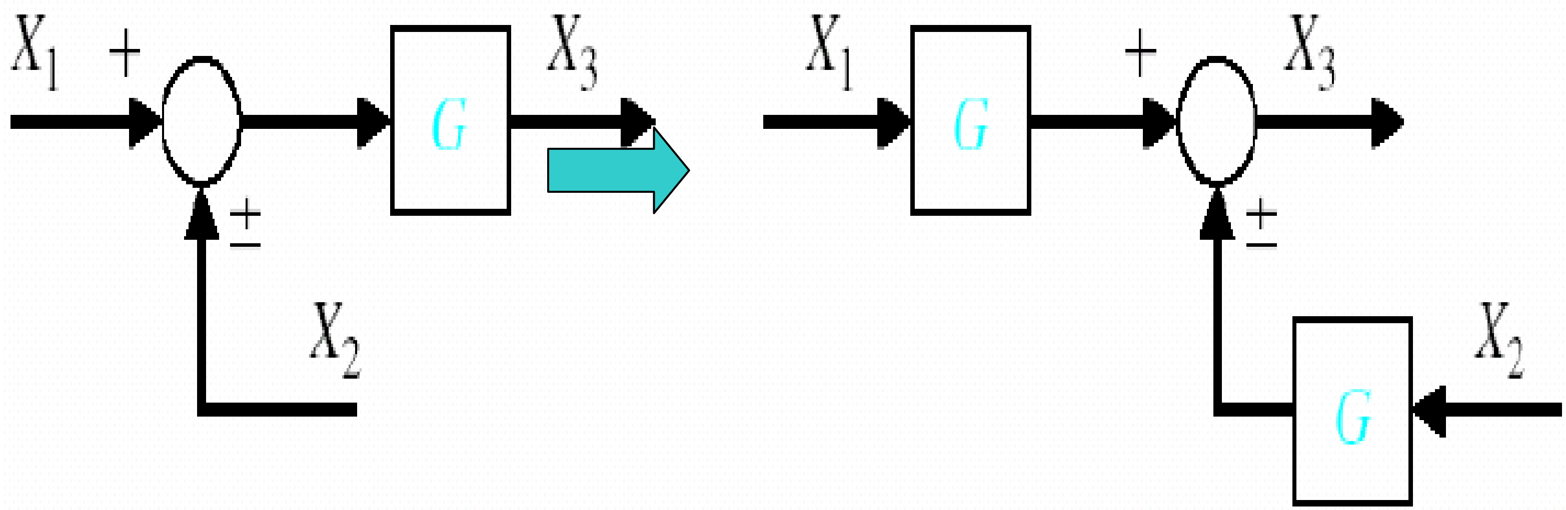
Block Diagram Transformations



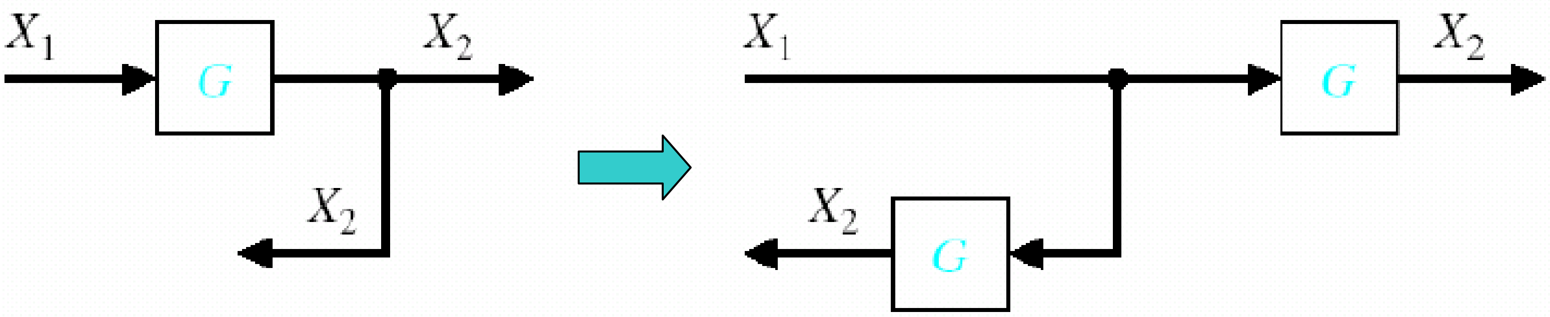
$$X_3 = G_2 X_2 = G_2(G_1 X_1) = G_2 G_1 X_1$$

Block Diagram Transformations



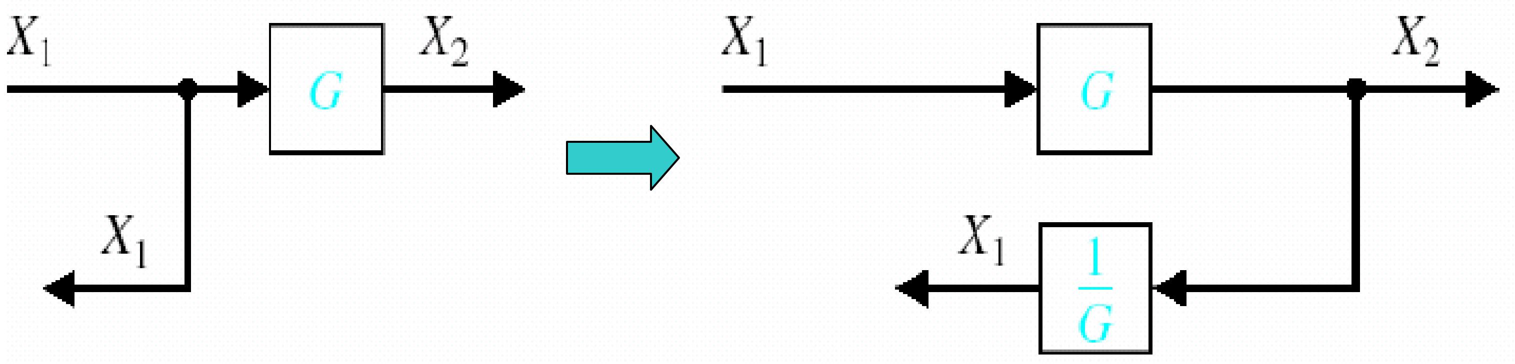
$$X_3 = G(X_1 \pm X_2) = GX_1 \pm GX_2$$

Block Diagram Transformations

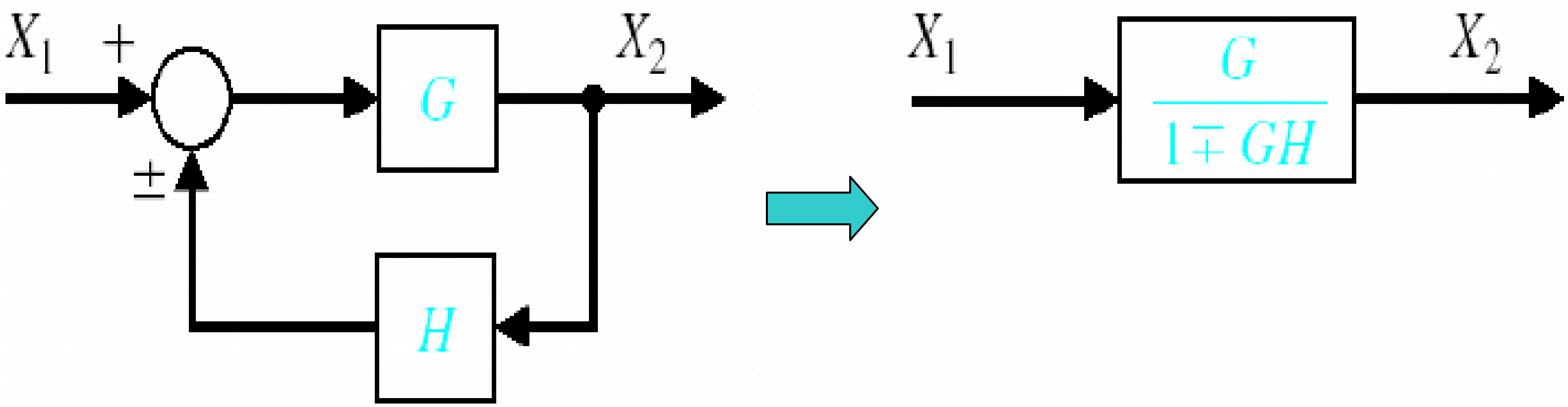


$$X_2 = GX_1$$

Block Diagram Transformations



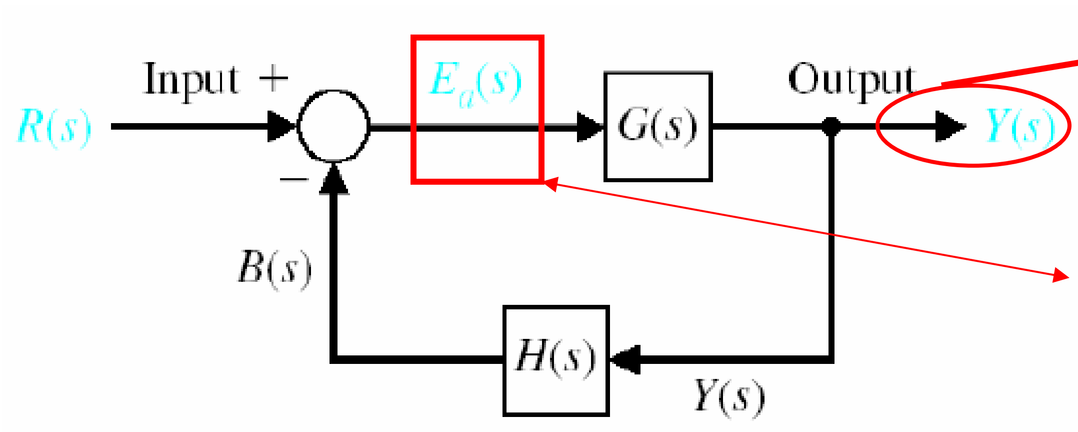
Block Diagram Transformations



$$X_2 = G(X_1 \pm HX_2)$$

$$(1 \mp GH)X_2 = GX_1$$

The Closed-Loop Transfer Function



$$Y(s) = G(s)E_a(s),$$

$$E_a(s) = R(s) - B(s)$$

$$= R(s) - H(s)Y(s).$$

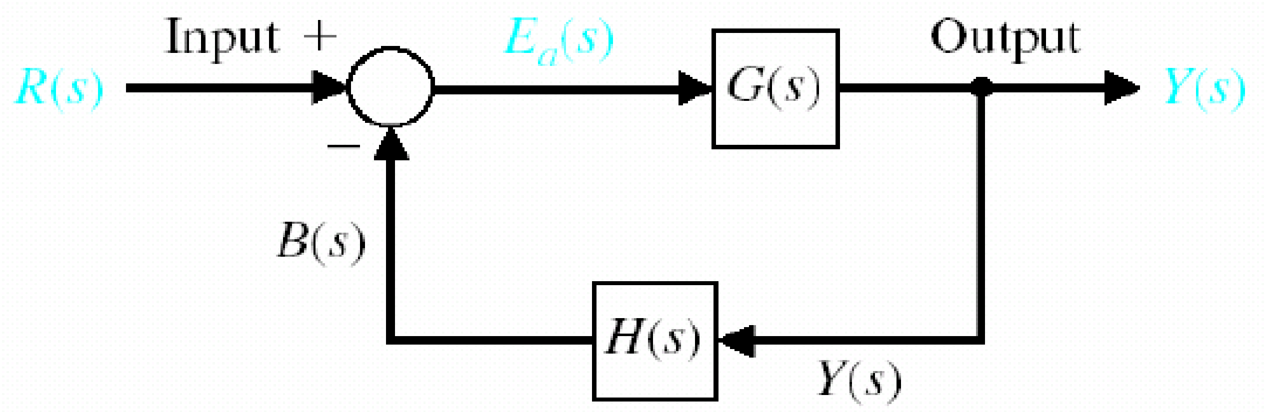
$$Y(s) = G(s)E_a(s) = G(s)[R(s) - H(s)Y(s)].$$

$$Y(s)[1 + G(s)H(s)] = G(s)R(s).$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

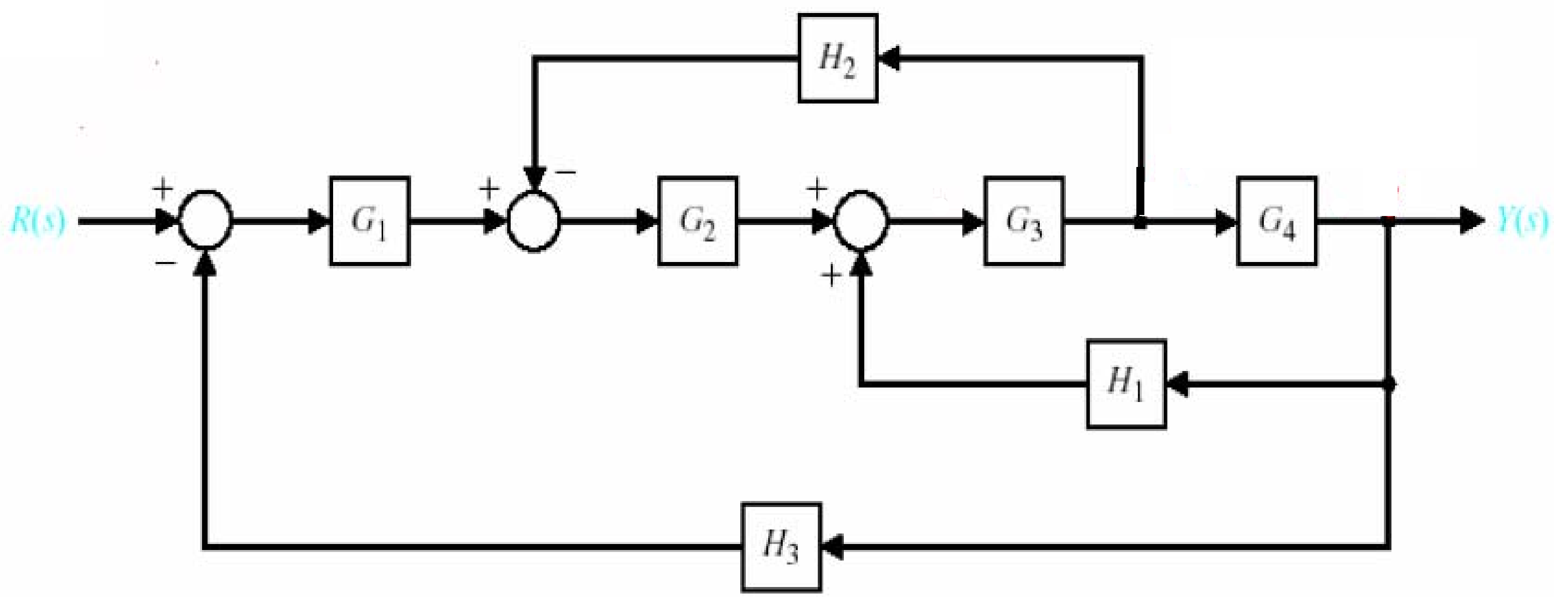
- Note: when $H(s) \neq 1 \Rightarrow E_a(s) \neq E(s)$ $\quad E(s) = R(s) - Y(s)$

The Closed-Loop Transfer Function



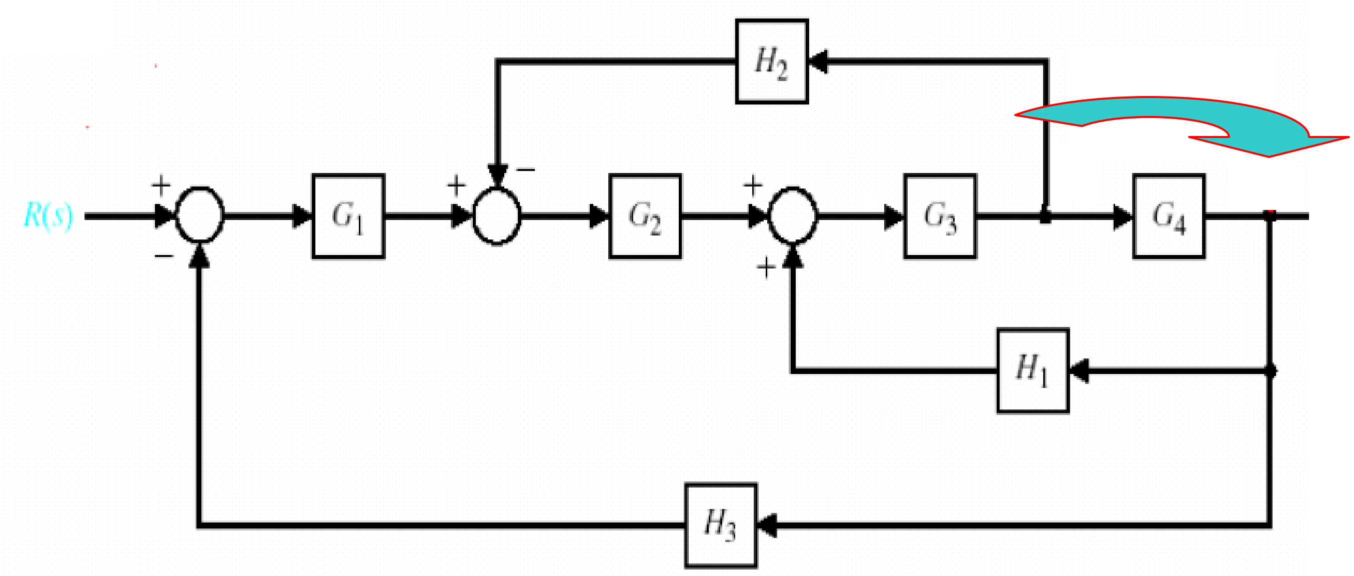
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = T(s)$$

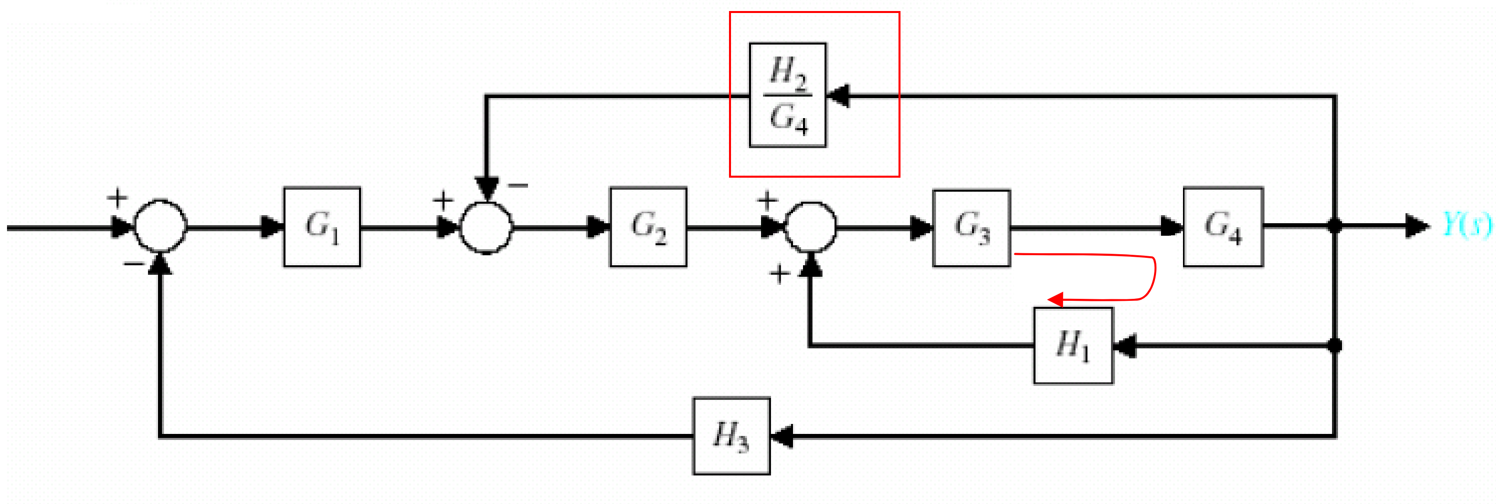
Block Diagram Reduction



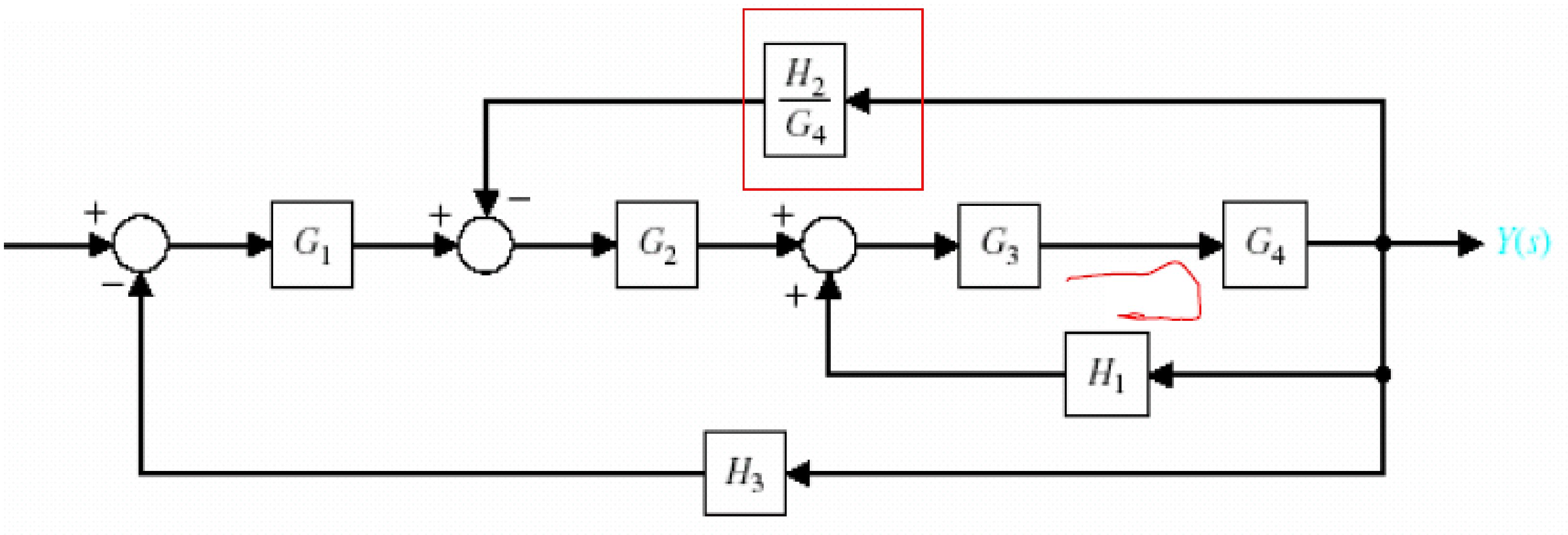
The block diagram is a visual representation of a set of linear algebraic equations relating the internal variables and inputs to the output variables. Solving by block diagram reduction, or Mason's Rule, must therefore be equivalent to solving a set of algebraic equations. The only special feature is that, generally, the internal; variables are coupled mostly to their neighbours as you move from left to right through the block diagram.

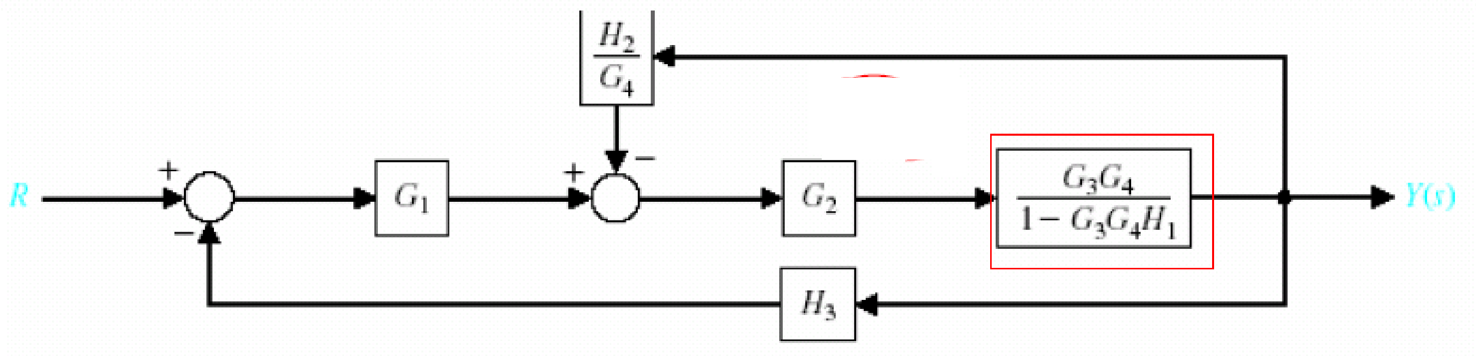
Block Diagram Reduction



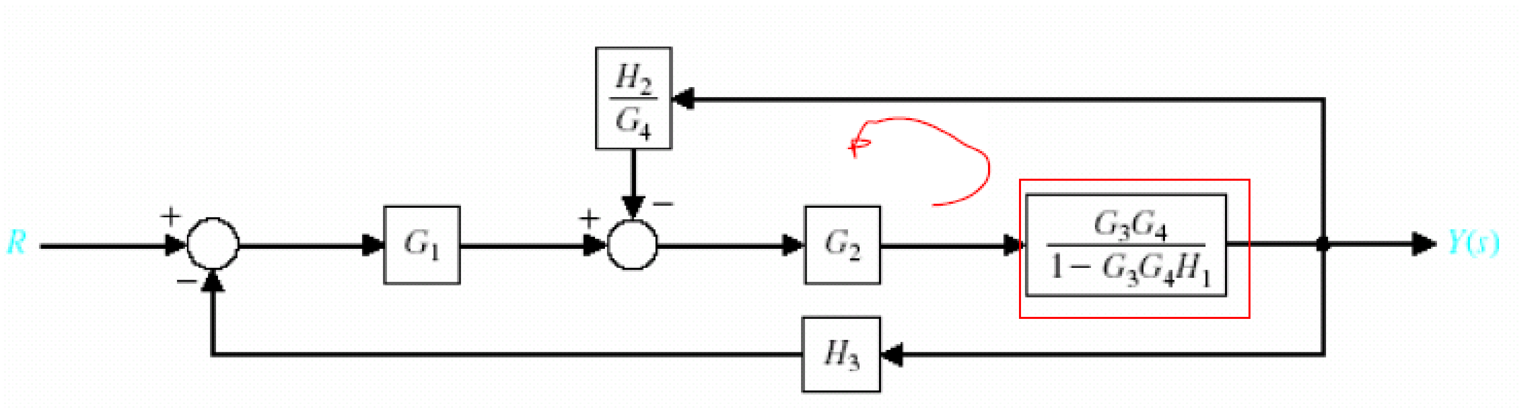


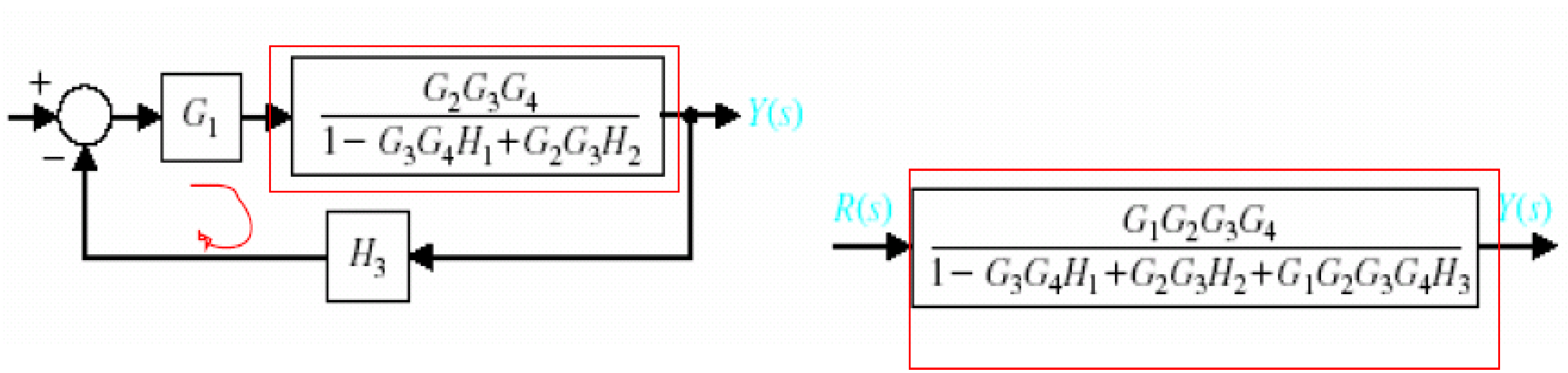
Block Diagram Reduction





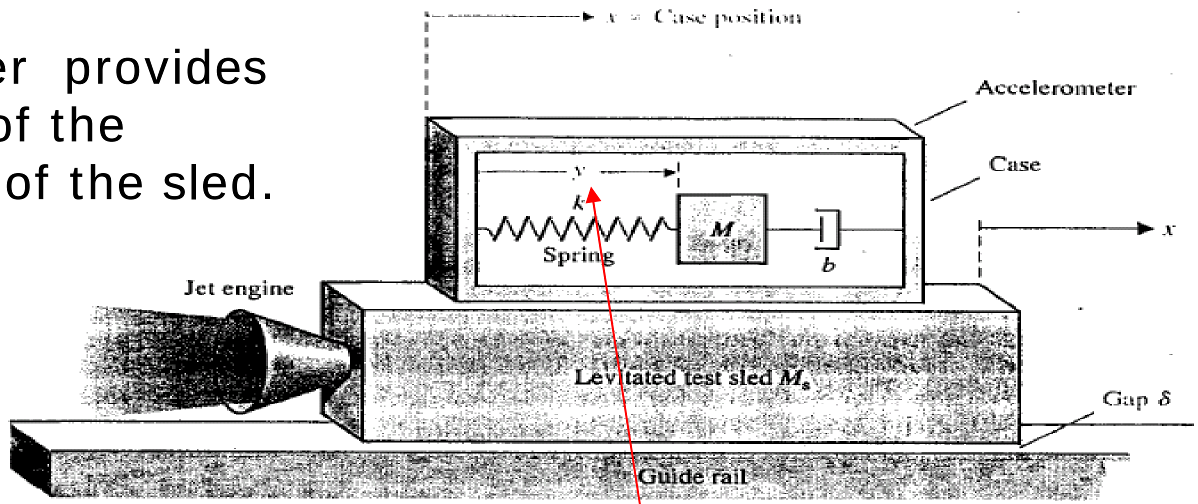
Block Diagram Reduction





Example 2.13 Mechanical accelerometer

- A mechanical accelerometer is used to measure the acceleration of a levitated test sled.
- The test sled is magnetically levitated above a guide rail a small distance δ.
- The accelerometer provides a measurement of the acceleration *a*(*t*) of the sled.



- The goal is to design an accelerometer with *an appropriate time* for the desired measurement characteristic, y(*t*)=*q a*(t), to be attained (*q* is a constant).

Obtain the ODE for the system

$$-b\frac{dy}{dt} - ky = M\frac{d^2y}{dt^2}$$

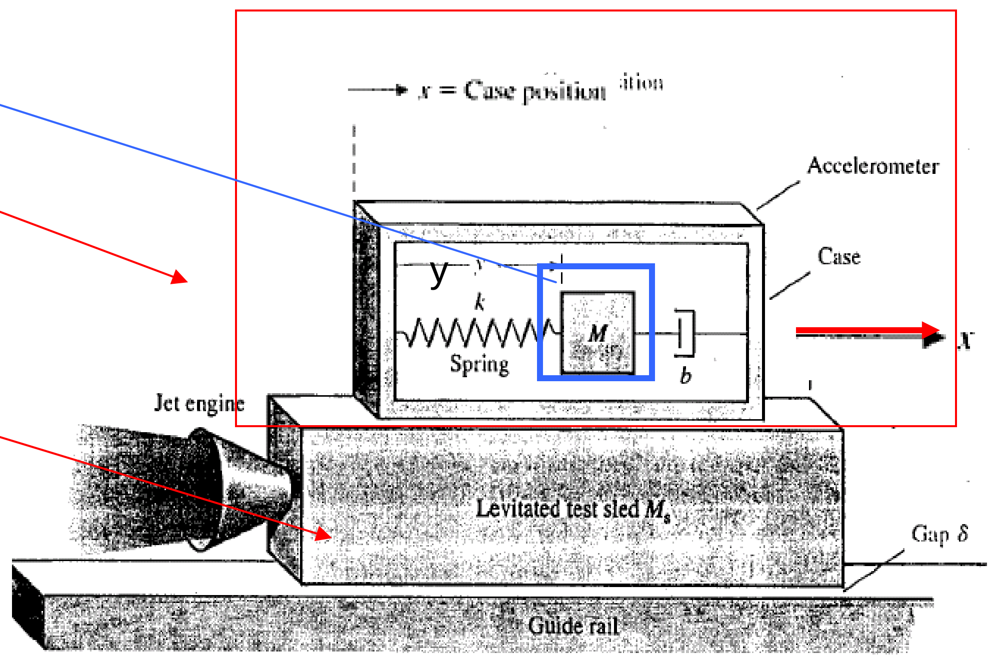
$$-b\frac{dy}{dt} - ky = M\frac{d^2}{dt^2}(y + x)$$

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = -M\frac{d^2x}{dt^2}.$$

$$M_s\frac{d^2x}{dt^2} = F(t), \Rightarrow \frac{d^2x}{dt^2} = \frac{1}{M_s}F(t),$$

$$M\ddot{y} + b\dot{y} + ky = -\frac{M}{M_s}F(t)$$

$$\ddot{y} + \frac{b}{M}\dot{y} + \frac{k}{M} = -\frac{F(t)}{M_s}$$



We select the coefficients where $b/M = 3, \; k/M = 2, \; F(t)/M_s = Q(t)$

Laplace transformation is used to obtain the solution

Laplace transfer with the initial conditions:

$$y(0) = -1, \quad \dot{y}(0) = 2$$

$$(s^2Y(s) - sy(0) - \dot{y}(0)) + 3(sY(s) - y(0)) + 2Y(s) = -\frac{P}{s}.$$

$$(s^2Y(s) + s - 2) + 3(sY(s) + 1) + 2Y(s) = -\frac{P}{s}.$$

$$(s^2 + 3s + 2)Y(s) + (s + 1) = -\frac{P}{s}$$

$$(s^2 + 3s + 2)Y(s) = -(s+1) - \frac{P}{s} = -\frac{s^2 + s + P}{s}$$

$$Y(s) = \frac{-(s^2 + s + P)}{s(s^2 + 3s + 2)} = \frac{-(s^2 + s + P)}{s(s+1)(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+2}$$

$$k_1 = sY(s)|_{s=0}, k_2 = (s+1).Y(s)|_{s=-1} \quad k_3 = (s+2)Y(s)|_{s=-2}$$

$$k_1 = \frac{-P}{2}, \quad k_2 = +P, \quad k_3 = \frac{-P-2}{2}$$

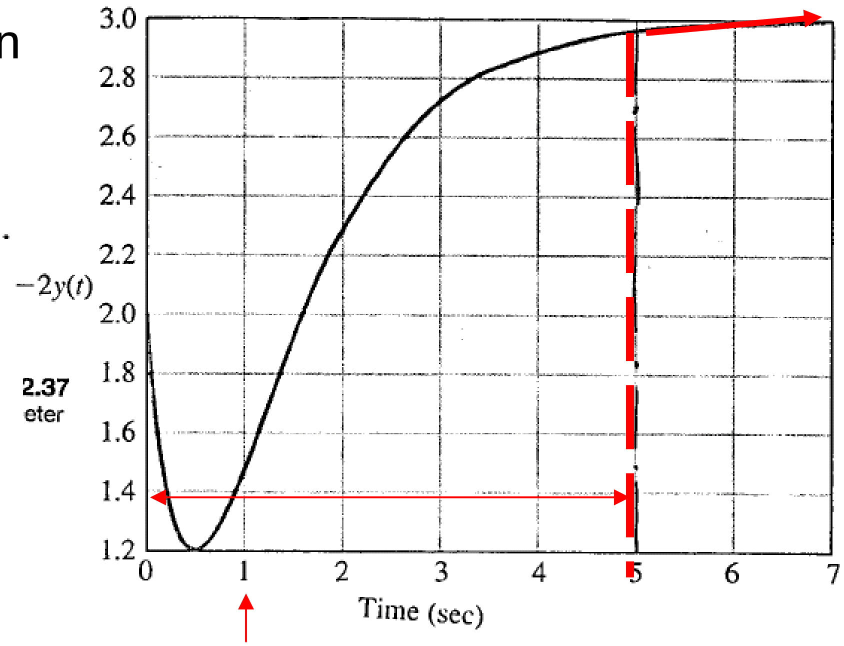
$$Y(s) = \frac{-(s^2 + s + P)}{s(s^2 + 3s + 2)} = \frac{-P}{2s} + \frac{P}{s+1} + \frac{-P-2}{2(s+2)}$$

Mechanical accelerometer

- Inverse Laplace transformation yield the output as

$$y(t) = \frac{1}{2}[-P + 2Pe^{-t} - (P+2)e^{-2t}], \quad t \geq 0.$$

A plot of $y(t)$ is shown in Fig. 2.27 for $P = 3$



- We can see that $y(t)$ is proportional to the magnitude of the force after 5 seconds.
- If this period is excessively long we must increase the spring constant, and the friction and reduce the mass.
- If we select the components so that $b/M = 12,$ and $k/M = 32,$ the accelerometer will attain the proportional response in one second.