Faculty of Engineering

DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING

EENG428 Introduction to Robotics

Instructor:

M. K. Uyguroğlu

Midterm EXAMINATION

April 19, 2016

Duration : 90 minutes

Number of Problems: 3

Good Luck

STUDENT'S	
NUMBER	
NAME	SOLUTIONS
SURNAME	

Problem		**Points**
1		30
2		40
3		30
TOTAL		100

Problem 1

For the figure shown in Fig. P1, find the homogeneous transformation matrices ${}^{i-1}\mathbf{A}_i$ and ${}^{0}\mathbf{A}_i$ for $i = 1, 2$ between the coordinate frames.

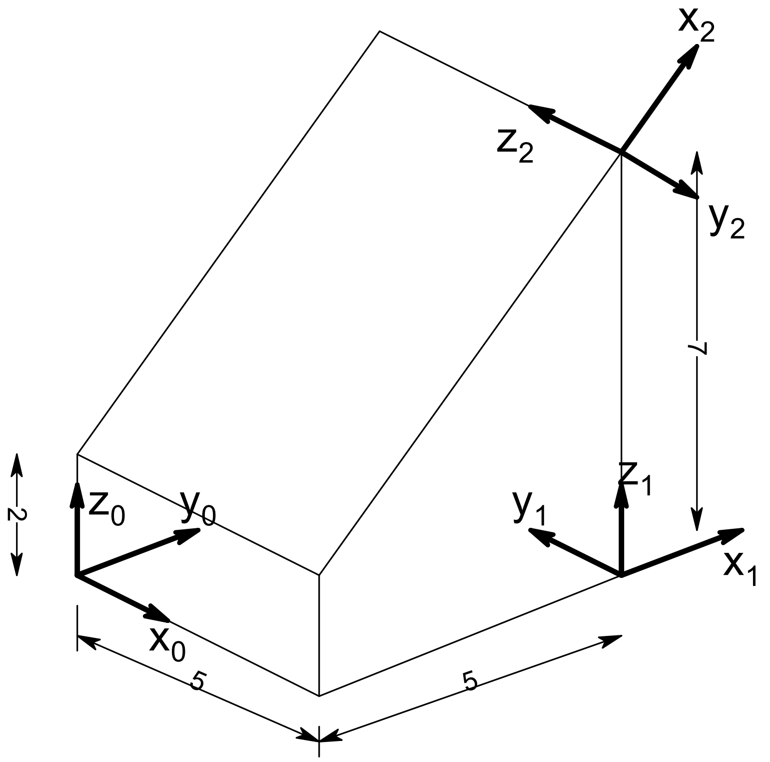

Figure P 1

$$
{}^{0}A_1 = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}A_2 = \begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.707 & -0.707 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}A_2 = A_1{}^{1}A_2 = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.707 & -0.707 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 5 \\ 0.707 & 0.707 & 0 & 5 \\ 0.707 & -0.707 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Problem 2

For the 3-DOF RRP robot shown in Fig.P2:

a) Assign appropriate frames for the Denavit-Hartenberg (D-H) representation.
b) Fill out the parameters Table.
c) Write all the A matrices.
d) Derive the forward kinematic equations for the robot.

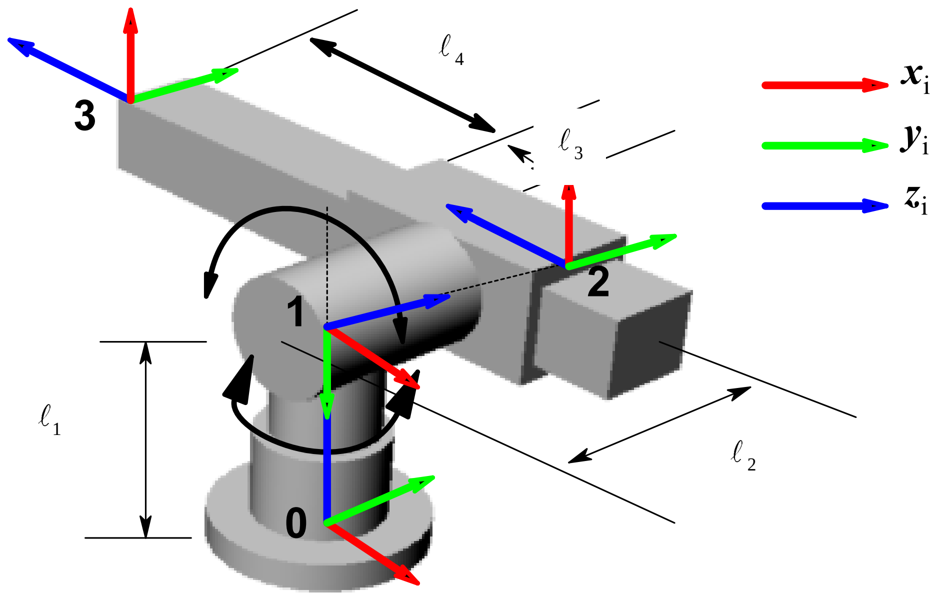

Figure P 2 Three-Link PRR manipulator

GIVEN:

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

D-H Parameters List:

Link_i	θ_i	d_i	a_i	α_i
1	θ_1^*	ℓ_1	0	-90
2	$\theta_2^* - 90$	ℓ_2	0	90
3	0	$\ell_3 + \ell_4^*$	0	0

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ -\cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + \ell_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = A_1 A_2 A_3$$

$${}^0T_H = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & \ell_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ -\cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & \ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + \ell_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} C_1 S_2 & -S_1 & -C_1 C_2 & -\ell_2 S_1 \\ S_1 S_2 & C_1 & -S_1 C_2 & \ell_2 C_1 \\ C_2 & 0 & S_2 & \ell_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3 + \ell_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} C_1 S_2 & -S_1 & -C_1 C_2 & -\ell_2 S_1 - (\ell_3 + \ell_4) C_1 C_2 \\ S_1 S_2 & C_1 & -S_1 C_2 & \ell_2 C_1 - (\ell_3 + \ell_4) S_1 C_2 \\ C_2 & 0 & S_2 & \ell_1 + (\ell_3 + \ell_4) S_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 3

D-H Parameters Table for the 3 DOF RPR manipulator is given in Table P3.

Table P3: D-H Parameters Table

Link$_i$	θ_i	d_i	a_i	α_i
1	θ_1^*	$\ell_1+\ell_2$	0	90
2	0	$\ell_3+\ell_4^*$	0	0
3	θ_3^*	ℓ_5	0	0

* joint variable

If $\ell_1=\ell_2=\ell_3=2,\ \ \ell_5=1$ and the hand frame relative to the base coordinate frame ${}^0T_H$ is given as:

$$
{}^0T_H=\begin{bmatrix} \frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{1}{2} & \frac{9}{4} \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{2} & \frac{-9\sqrt{3}}{4} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

then determine the joint variables $\theta_1, \ell_4,$ and θ_3.

GIVEN

Some General Analytical Inverse Kinematics Formulas	
IF	THEN
$\cos\theta = b$	$\theta = A\tan 2\left(\pm\sqrt{1-b^2}, b\right)$;i.e., both θ and -θ
$\sin\theta = a$	$\theta = A\tan 2\left(a, \pm\sqrt{1-a^2}\right)$;i.e., both θ and (180-θ)
$\sin\theta = a$ $\cos\theta = b$	$\theta = A\tan 2(a,b)$
$a\cos\theta - b\sin\theta = 0$	$= A\tan 2(a,b)$ *and* $\theta = A\tan 2(-a,-b)$ } i.e., both θ and ($\theta \pm 180$)

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & \ell_1+\ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3+\ell_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & \ell_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = A_1 A_2 A_3$$

$${}^0T_H = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & \ell_1+\ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \ell_3+\ell_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & \ell_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & (\ell_3+\ell_4)\sin\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & -(\ell_3+\ell_4)\cos\theta_1 \\ 0 & 1 & 0 & \ell_1+\ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & \ell_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_H = \begin{bmatrix} C_1C_3 & -C_1S_3 & S_1 & \ell_5 S_1 + (\ell_3+\ell_4)S_1 \\ S_1C_3 & -S_1S_3 & -C_1 & -\ell_5 C_1 - (\ell_3+\ell_4)C_1 \\ S_3 & C_3 & 0 & \ell_1+\ell_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left.\begin{aligned} \sin\theta_1 &= \frac{1}{2} \\ \cos\theta_1 &= \frac{\sqrt{3}}{2} \end{aligned}\right\} \quad \theta_1 = a\tan 2\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = 30^0$$

$$\left.\begin{array}{l}\sin\theta_3 = \dfrac{\sqrt{3}}{2} \\ \cos\theta_3 = \dfrac{1}{2}\end{array}\right\} \quad \theta_3 = a\tan 2\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = 60^0$$

$$(\ell_3 + \ell_4 + \ell_5) S_1 = \frac{9}{4}$$

$$(3 + \ell_4) = \frac{9}{4 \times S_1} = \frac{9}{4 \times \frac{1}{2}} = 4.5$$

$$\boxed{\ell_4 = 4.5 - 3 = 1.5}$$