EASTERN MEDITERRANEAN UNIVERSITY  
FACULTY OF ARTS AND SCIENCES  
DEPARTMENT OF PHYSICS  

PHYS 101 – MIDTERM EXAM  
2017-2018 Spring (18 April 2018)  

Student Number | Name | Surname | Group | Signature
---|---|---|---|---
SOLUTION KEY

<table>
<thead>
<tr>
<th>Position Vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$</th>
<th>Displacement $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$</th>
<th>Average Speed $v_{avg} = \frac{\text{total distance}}{\text{time elapsed}}$</th>
<th>Average Velocity $v = \frac{\Delta\mathbf{r}}{\Delta t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Acceleration $\mathbf{a}_{avg} = \frac{\Delta\mathbf{v}}{\Delta t}$</td>
<td>Instantaneous Speed $v =</td>
<td>\mathbf{v}</td>
<td>= \sqrt{v_x^2 + v_y^2 + v_z^2}$</td>
</tr>
</tbody>
</table>

Equations for Linear Motion with Constant Acceleration:
- The equation of position, $\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2$
- The equation of velocity, $\mathbf{v} = \mathbf{v}_i + \mathbf{a} t$
- $\sum \mathbf{F} = m\mathbf{a}$
- $a = \frac{v^2}{r}$
- $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$
- $\mu_f \leq \mu_s \leq \frac{f_{static}}{mg}$
- $f_{kinetic} = \mu_k F_N$
- $a = \frac{v^2}{r}$
- $g = 9.8 \text{ m/s}^2$

The solution methods for the problems of this exam are limited to the formulae given in the above table. Thus, using any other formula different than formulae given above will cause to losing points, even if your answers are correct!

### Total (36p)

<table>
<thead>
<tr>
<th>P1 (7p)</th>
<th>P2 (6p)</th>
<th>P3 (8p)</th>
<th>P4 (8p)</th>
<th>P5 (7p)</th>
<th>TOTAL (36p)</th>
</tr>
</thead>
</table>

1) Answer the followings by using the data in the given Figure.

a) Find the magnitudes and directions of the vectors given in the Figure.

(2p)

**SLN:**

- magnitude of $\mathbf{A}$, $|\mathbf{A}| = 5 \text{ m}$; direction of $\mathbf{A}, \theta_A = 55^\circ$
- magnitude of $\mathbf{B}$, $|\mathbf{B}| = 5 \text{ m}$; direction of $\mathbf{B}, \theta_B = 180 - 33 = 147^\circ$

b) Write the vectors in unit vector notation. (2p)

**SLN:**

\[
\mathbf{A} = (5\cos 55^\circ \mathbf{i} + 5\sin 55^\circ \mathbf{j}) = (2.87 \mathbf{i} + 4.10 \mathbf{j}) \text{ m}
\]

\[
\mathbf{B} = (2\cos 147^\circ \mathbf{i} + 2\sin 147^\circ \mathbf{j}) = (-1.68 \mathbf{i} + 1.09 \mathbf{j}) \text{ m}
\]

c) If it is known that $\left\{ \frac{1}{2} \mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 0 \right\}$, find the magnitude and direction, the angle it makes with $+x$ axis, of vector $\mathbf{C}$. (3p)

**SLN:**

\[
\left\{ \frac{1}{2} \mathbf{A} - 2\mathbf{B} + 3\mathbf{C} = 0 \right\} \Rightarrow \left\{ \frac{1}{2} (2.87 \mathbf{i} + 4.10 \mathbf{j}) - 2(-1.68 \mathbf{i} + 1.09 \mathbf{j}) + 3\mathbf{C} \right\} = 0
\]

\[-3\mathbf{C} = 1.44 \mathbf{i} + 2.05 \mathbf{j} + 3.36 \mathbf{j} - 2.18 \mathbf{j} = 4.80 \mathbf{i} - 0.13 \mathbf{j} \Rightarrow \mathbf{C} = \frac{4.80 \mathbf{i} - 0.13 \mathbf{j}}{-3} = (-1.60 \mathbf{i} + 0.04 \mathbf{j}) \text{ m}
\]
2) The position of a particle moving in xy-plane is given as \( x = (3t^2 + 2t)m \) and \( y = (2t^3 - t)m \), where \( t \) is in seconds.

a) Write the position vector, \( \vec{r} \), of the particle when \( t = 2s \). (2p)

**SLN:**
\[
\vec{r}(t) = x\hat{i} + y\hat{j} = (3t^2 + 2t)\hat{i} + (2t^3 - t)\hat{j}m
\]
\[ \vec{r}(2) = \{(3(2)^2 + 2(2))\hat{i} + (2(2)^3 - 2)\hat{j}\} = \{(12 + 4)\hat{i} + (16 - 2)\hat{j}\} = (16\hat{i} + 14\hat{j})m \]

b) Find the velocity vector, \( \vec{v} \), of the particle when \( t = 2s \). (2p)

**SLN:**
\[
\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}((3t^2 + 2t)\hat{i} + (2t^3 - t)\hat{j}) = ((6t + 2)\hat{i} + (6t^2 - 1)\hat{j})m/s
\]
\[ \vec{v}(2) = ((6(2) + 2)\hat{i} + (6(2)^2 - 1)\hat{j}) = \vec{v}(t) = ((12 + 2)\hat{i} + (24 - 1)\hat{j}) = (14\hat{i} + 23\hat{j})m/s \]

c) Find the acceleration vector, \( \vec{a} \), of the particle when \( t = 2s \). (2p)

**SLN:**
\[
\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}((6t + 2)\hat{i} + (6t^2 - 1)\hat{j}) = (6\hat{i} + (12t)\hat{j})m/s^2
\]
\[ \vec{a}(2) = ((8)\hat{i} + (12(2))\hat{j}) = \vec{a}(t) = (6\hat{i} + 24\hat{j})m/s^2 \]

3) A stone is projected at a building of height \( h \) with an initial speed 50m/s directed at angle \( \theta = 57^\circ \) above the horizontal. The stone strikes at \( A \) 6s after launching.

a) Find the height \( h \) of the cliff. (2p)

**SLN:**
\[
\vec{r} = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2
\]
\[ (xt + y) = 0 + (50\cos57t + 50\sin57)t + \frac{1}{2}(-9.8t^2) = (27.23t + 41.93)t - 4.9t^2 \]
\[ x = 27.23t ; \quad h = 41.93 - 4.9t^2 \]
\[ h = 41.93 - 4.9t^2 = (41.93)(6) - (4.9)(6)^2 = 251.6 - 176.4 = 75.2m \]

b) Find the speed of the stone just before it hits point \( A \). (2p)

**SLN:**
\[
\vec{v} = \vec{v}_i + \vec{a} t = (27.23t + 41.93) + (-9.8)(6) = (27.23t - 16.87)t m/s
\]
speed, \( |\vec{v}| = \sqrt{(27.23)^2 + (-16.87)^2} = 32 m/s \)

c) Find the maximum height \( H \) reached above the ground. (4p)

**SLN:**
\[
\vec{r} = \vec{r}_i + \vec{v}_i t + \frac{1}{2} \vec{a} t^2
\]
\[ (xt + y) = 0 + (50\cos57t + 50\sin57)t + \frac{1}{2}(-9.8t^2) = (27.23t + 41.93)t - 4.9t^2 \]
\[ (H) = (50\sin57)t + \frac{1}{2}(-9.8t^2) = (41.93)t - 4.9t^2 \quad \text{so } t \text{ for maximum height is needed} \]
\[ \vec{v} = \vec{v}_i + \vec{a} t \Rightarrow (27.23) = (27.23t + 41.93) + (-9.8)(t) \]
\[ 9.8t = 41.93 \]
\[ t = 4.29s \]
\[ H = (41.93)(4.29) - (4.9)(4.29)^2 = 179.88 - 90.18 = 89.90m \]
4) A force of magnitude $|\vec{F}_A| = 30N$ and $\theta = 30^\circ$ is applied to a box which is on a rough surface as given in Figure. The coefficients of static and kinetic friction between the box and the surface are $\mu_s = 0.2$ and $\mu_k = 0.1$ respectively. What is the type of the friction force acting on the box and its magnitude for the following cases.

a) When the mass of the box is $m = 50kg$. (4p)

\[ f_{\text{max}} = \mu_s F_N \]
\[ \sum \hat{F}_x = \vec{F}_{Ax} = 30 \cos 30 = 26N \text{ is the net force in the direction of the motion.} \]
\[ f(\text{max}) = \mu_s F_N = (0.2)(9.8)(50) + 15 = 101N \]
\[ f(\text{max}) > F_{\text{net}} \text{ will not move} \]
\[ \text{so static friction} \]
\[ f_s = 26N \]

b) When the mass of the box is $m = 10kg$. (4p)

\[ f(\text{max}) = \mu_s F_N = (0.2)(9.8)(10) + 15 = 22.6N \]
\[ f(\text{max}) < F_{\text{net}} \text{ move} \]
\[ \text{so kinetic friction} \]
\[ f_k = \mu_k F_N = (0.1)(9.8)(10) + 15 = 11.3N \]

5) A mass of $2m$ is connected to two masses $m$ and $3m$ as shown in the Figure. The strings connecting the masses are light. The pulleys and all the surfaces are frictionless.

a) Draw separate free body diagrams for each of the masses, taking care to identify all forces acting. (3p)

b) If the angle of the slope $\theta = 30^\circ$, and the system is released from rest, does the hanging mass $2m$ remain where it is, accelerate upwards or accelerate downwards? (4p)

\[ F_{g2} (= 2mg) \text{ is opposed by } T_1 (= \vec{F}_{g1x} = 0.5mg, \text{ in equilibrium condition}) \text{ and;} \]
\[ T_2 (= \vec{F}_{g3x} = 1.5mg, \text{ in equilibrium condition}) \]
These two tensions adds up to $2mg$ which is exactly balancing $2m$ having a weight of $2mg$. Thus, the system is stable, not moving.