W = \int_{t_i}^{t_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta \vec{r} = |\vec{F}| |\Delta \vec{r}| \cos \theta \quad W_F = -fd \quad p = \frac{\sum W}{t} \quad K = \frac{1}{2}mv^2 \quad U_g = mgh \quad U_k = \frac{1}{2}k(\Delta x)^2

E_{mech} = K + U \quad \left( \sum E_{mech} \right)_t + \left( \sum W_{ext} \right)_{t_f} = \left( \sum E_{mech} \right)_f \quad \vec{p} = m\vec{v} \quad \Delta \vec{p} = \vec{I} \quad g = 9.8 m/s^2

I = \int_{t_i}^{t_f} \vec{F} dt (Impulse) \quad \text{Impulse of constant Force,} \quad \vec{I} = \vec{F}_{avg}\Delta t \quad \left( \sum m_i\vec{v}_i \right)_{before} = \left( \sum m_i\vec{v}_i \right)_{after}

The solution methods for the problems of this exam are limited to the formulae given in the above table. Thus, using any other formula different than formulae given above will cause to losing points, even if your answers are correct!

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Problem 1: A particle of mass \( m = 0.001 kg \) moving in the \( xy \)-plane undergoes a displacement given by \( \Delta \vec{r} = (3\vec{f} - 2\vec{f})m \) as a constant force \( \vec{F} = (10\vec{f} - 4\vec{f})N \) acts on the particle.

a) Find the work done by the force on the particle. (2p)

**SLN:**

\[ W = \vec{F} \cdot \Delta \vec{r} = [(10\vec{f} - 4\vec{f})\cdot [(3\vec{f} - 2\vec{f})] = 30 + 8 = 38 Joules \]

b) Find the work done by gravity on the particle. (2p)

**SLN:**

\[ W = \vec{F}_g \cdot \Delta \vec{r} = [(-m\vec{g})] \cdot [(3\vec{f} - 2\vec{f})] = -((0.001)(9.8)) \cdot [(3\vec{f} - 2\vec{f})] = 0.0196 Joules \]
**Problem 2:** In the figure, a block of mass \( m = 1kg \) is pushed against a spring of spring constant \( k = 100 N/m \) compressing the spring and held there. When the block is released from rest, the block moves along the track from one level to a higher level after passing through an intermediative valley. The track is frictionless until the block reaches the higher level. There, a frictional force stops the block in a distance \( d \). The initial compression on the spring is \( \Delta x = 0.3m \), the height difference \( h \) is \( 0.4m \), and \( \mu_k \) is 0.2. Find \( d \). (8p)

\[
\begin{align*}
\text{SLN:} \\
\left( \sum E_{\text{mech}} \right)_i + \left( \sum W_{\text{ext}} \right)_{i \rightarrow f} &= \left( \sum E_{\text{mech}} \right)_f \\
K_i + U_{gi} + U_{ei} + (-fd) &= K_f + U_{gf} + U_{ef} \\
0 + 0 + \frac{1}{2}k(\Delta x)^2 - \mu_k F_N d &= 0 + mgh + 0 \\
0 + 0 + \frac{1}{2}(100)(0.3)^2 - (0.2)mg(d) &= 0 + (1)(9.8)(0.4) \\
4.5 - (0.2)(1)(9.8)(d) &= 3.92 \\
\Rightarrow 1.96d &= 0.58 \\
\Rightarrow d &= 0.3m
\end{align*}
\]

**Problem 3:** In the overhead view of the Figure, a 0.4kg ball with a speed of \( v \) of \( 5m/s \) strikes a ceiling at an angle \( \theta \) of \( 40^\circ \) and then rebounds with the same speed and angle. It is in contact with the wall for 0.01s.

a) Determine the momentum of the ball just before it strikes the ceiling. (2p)

\[
\begin{align*}
\vec{p}_i &= m\vec{v}_i = (0.4)(5\cos 140\hat{f} + 5\sin 140\hat{f}) = (-1.53\hat{f} + 1.28\hat{f})kgm/s^2
\end{align*}
\]

b) Determine the momentum of the ball just after it strikes the ceiling. (2p)

\[
\begin{align*}
\vec{p}_f &= m\vec{v}_f = (0.4)(5\cos 220\hat{f} + 5\sin 220\hat{f}) = (-1.53\hat{f} - 1.28\hat{f})kgm/s^2
\end{align*}
\]

c) Determine the impulse on the ball from the ceiling in unit vector notation. (2p)

\[
\vec{I} = \vec{p}_f - \vec{p}_i = (-1.53\hat{f} - 1.28\hat{f}) - (-1.53\hat{f} + 1.28\hat{f}) = (-2.56\hat{f})
\]

d) Determine the magnitude of the average force exerted by the ceiling on the ball. (2p)

\[
\begin{align*}
\vec{F}_{avg} &= \vec{I} / \Delta t = (-2.56\hat{f}) / 0.01 = -256\hat{f}N \\
\vec{F}_{avg} &= 256N
\end{align*}
\]
LABORATORY EXAM

**QUESTION1:** Assume that you are performing an experiment by using a force table. You are given two forces with magnitudes $|\vec{F}_1| = 70N$ and $|\vec{F}_2| = 50N$ in the directions illustrated on the figure below. Calculate the magnitude and direction of the force, $\vec{F}_3$, balancing those two given forces and draw it on the given figure. (*Hint*: $(\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0)$) (5p)

### Solution (SLN):

\[ \vec{F} = |\vec{F}| \cos \theta \hat{i} + |\vec{F}| \sin \theta \hat{j} \]

\[ \vec{F}_1 = 70 \cos 130^\circ \hat{i} + 70 \sin 130^\circ \hat{j} = (-45 \hat{i} + 53.6 \hat{j}) N \]

\[ \vec{F}_2 = 50 \cos 240^\circ \hat{i} + 50 \sin 240^\circ \hat{j} = (-25 \hat{i} - 43.3 \hat{j}) N \]

\[ \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \]

\[ (-45 \hat{i} + 53.6 \hat{j}) + (-25 \hat{i} - 43.3 \hat{j}) + \vec{F}_3 = 0 \]

\[ (-70 \hat{i} + 10.3 \hat{j}) + \vec{F}_3 = 0 \]

\[ \vec{F}_3 = (70 \hat{i} - 10.3 \hat{j}) N \]

\[ |\vec{F}_3| = \sqrt{(70^2 + 10.3^2)} = 70.75 N \]

\[ \theta_{\vec{F}_3} = \tan^{-1} \left( \frac{10.3}{-70} \right) = -8.37^\circ = 351.63^\circ \]
**QUESTION2:** Assume that you are performing an experiment about projectile motion. Consider that, in order to understand the principles of projectile motion, you throw a small steel ball twice with different initial speeds and throwing angles. What is your expectation about the relation between the accelerations of the ball during those two projectile motions? Conclude about your answer. (3p)

*SLN:* 

The acceleration of any projectile motion is independent from all other physical parameters and it is \( g = -9.8 \text{ m/s}^2 \).

**QUESTION3:** Assume that you are performing an experiment about work and energy by using the set-up given below. You release a car of mass \( m = 0.2 \text{ kg} \) from point A, which is above 0.5m from the table-top. When the car reaches at point B, which is 0.4m above the table-top, a sensor reads the speed of the car as \( 0.8 \text{ m/s} \).

![Diagram of the experiment setup](image)

a) What is the cars initial total mechanical energy with respect to table-top. (1p)

*SLN:* 

\[ (E_{\text{mech}})_A = K_A + U_A = \frac{1}{2}mv_A^2 + mgh_A = 0 + (0.2)(9.8)(0.5) = 0.98J \]

b) What is the cars final total mechanical energy energy with respect to table-top. (2p)

*SLN:* 

\[ (E_{\text{mech}})_B = K_B + U_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}(0.2)(0.8)^2 + (0.2)(9.8)(0.4) = 0.85J \]

c) Did the total mechanical energy of the car conserve between points A and B? If not what is the reason? (2p)

*SLN:* 

The total mechanical energy of the car was reduced, did not conserve. This is because of the friction forces affecting the car during the motion from A to B.