**PROBLEMS for ROTATION**

**PROBLEM 1:** The angular position of a point on the rim of a rotating wheel is given by \( \theta = 4t - 3t^2 + t^3 \), where \( \theta \) is in radians and \( t \) is in seconds.

a) what is the angular position at \( t = 2s \)?

b) what is the angular position at \( t = 2s \)?

c) what is the average angular velocity for the time interval from \( t = 2s \) to \( t = 4s \)?

d) what is the angular velocity at \( t = 2s \)?

e) what is the angular velocity \( t = 4s \)?

f) what is the average angular acceleration for the time interval from \( t = 2s \) to \( t = 4s \)?

g) what is the instantaneous acceleration at \( t = 2s \)?

**SLN:**

\[ \theta = (4t - 3t^2 + t^3) \text{rad} \]

\[ \omega = \frac{d\theta}{dt} = \frac{d}{dt} (4t - 3t^2 + t^3) = (4 - 6t + 3t^2) \text{rad/s} \]

\[ \alpha = \frac{d\omega}{dt} = \frac{d}{dt} (4 - 6t + 3t^2) = (-6 + 6t) \text{rad/s}^2 \]

a) \( \theta(t = 2s) = [(4)(2) - 3(2)^2 + (2)^3] = (8 - 12 + 8) = 4 \text{rad} \)

d) \( \omega(t = 2s) = [(4 - 6)(2) + (3)(2)^2] = (4 - 12 + 12) = 4 \text{ rad/s} \)

e) \( \omega(t = 4s) = [(4 - 6)(4) + (3)(4)^2] = (4 - 24 + 48) = 28 \text{ rad/s} \)

f) \( \alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\omega(t=4s) - \omega(t=2s)}{4-2} = \frac{32-4}{2} = 14 \text{ rad/s} \)

g) \( \alpha(t = 2s) = [-6 + (6)(2)] = (-6 + 12) = 6 \text{ rad/s}^2 \)
**PROBLEM 2:** A gridstone rotates at constant angular acceleration $\alpha = 0.35 \text{rad/s}^2$. At time $t = 0$, it has an angular velocity of $\omega_0 = -4.6 \text{rad/s}$ and a reference line on it is horizontal, at the angular position $\theta_0 = 0$.

a) At what time after $t = 0$ is the reference line at the angular position $\theta = 5 \text{rev}$?

b) At what time $t$ does the grindstone momentarily stop?

**SLN:**

a) $\theta = 5 \text{rev} = (5 \text{rev})(\frac{2\pi}{\text{rev}})(\frac{3.14 \text{rad}}{\pi}) = 31.4 \text{rad}$

\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \Rightarrow \quad 31.4 = 0 + (-4.6)t + \frac{1}{2}(0.35)t^2 \quad \Rightarrow \quad 0.175t^2 - 4.6t - 31.4 = 0 \quad \Rightarrow \quad t = 32 \text{s} \]

b) $\omega = \omega_0 + \alpha t \quad \Rightarrow \quad 0 = -4.6 + (0.35)t \quad \Rightarrow \quad t = 13 \text{s} $

**Problem 3:** An astronaut is tested in a centrifuge with radius $10 \text{m}$ and rotating according to $\theta = 0.3t^2$. At $t = 5 \text{s}$, what are the magnitudes of the a) angular velocity, b) linear velocity, c) tangential acceleration, and d) radial acceleration?

**SLN:**

a) $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(0.3t^2) = (0.6t) \frac{\text{rad}}{\text{s}} \quad \Rightarrow \quad \omega(t = 5 \text{s}) = (0.6)(5) = 3 \frac{\text{rad}}{\text{s}}$

b) $v = r\omega = (10)(0.6t) = 6t \frac{\text{m}}{\text{s}} \quad \Rightarrow \quad v(t = 5 \text{s}) = (6)(5) = 30 \frac{\text{m}}{\text{s}}$

c) $a_t = \frac{dv}{dt} = \frac{d}{dt}(6t) = 6 \frac{\text{m}}{\text{s}^2}$

d) $a_r = \frac{v^2}{r} = \frac{(6t)^2}{10} = 3.6t^2 \frac{\text{m}}{\text{s}^2} \quad \Rightarrow \quad a_r(t = 5 \text{s}) = (3.6)(5^2) = 90 \frac{\text{m}}{\text{s}^2}$
**Problem 4:** While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decreased the angular velocity of the cylinder from $3.4 \text{ rad/s}$ to $2 \text{ rad/s}$ in $20 \text{ rev}$, at constant angular acceleration.

a) What is the constant angular acceleration during this decrease in angular speed?

b) How much time did the speed decrease take?

**SLN:**

\[
\theta = 20 \text{ rev} = (20 \text{ rev}) \left( \frac{2\pi}{\text{rev}} \right) \left( \frac{3.14 \text{ rad}}{\pi} \right) = 125.6 \text{ rad}
\]

\[
\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \Rightarrow \quad 125.6 = 0 + (3.4)t + \frac{1}{2} \alpha t^2 \quad \text{ and } \quad \omega = \omega_0 + \alpha t \quad \Rightarrow \quad 2 = 3.4 + \alpha t \quad \Rightarrow \quad \alpha = \frac{2-3.4}{t} = \frac{-1.4}{t}
\]

\[
125.6 = 0 + (3.4)t + \frac{1}{2} \left( \frac{-1.4}{t} \right) t^2 \quad \Rightarrow \quad 125.6 = 3.4t - 0.7t \quad \Rightarrow \quad 125.6 = 2.7t \quad \Rightarrow \quad t = 46.52 \text{ s}
\]

\[
\therefore \quad \alpha = \frac{-1.4}{t} = \frac{-1.4}{46.52} = -0.03 \text{ rad/s}^2
\]

**Problem 5:** A flywheel turns through $40 \text{ rev}$ as it slows from an angular speed of $1.5 \text{ rad/s}$ to stop. Assume constant angular acceleration.

a) Find the time for the flywheel to come to rest. \((\text{ans: } t = 335 \text{ s})\)

b) What is its angular acceleration? \((\text{ans: } \alpha = -0.0045 \text{ rad/s}^2)\)

c) How much time is required for it to complete the first 20 of the 40 revolutions? \((\text{ans: } t = 98.2 \text{ s})\)
PROBLEM 6: Wheel $A$ of radius $r_A = 10 \text{cm}$ is coupled by belt $B$ to wheel $C$ of radius $r_C = 25 \text{cm}$, as shown in the Figure. The angular speed of wheel $A$ is increased from rest at a constant rate of $1.6 \text{rad/s}^2$. Find the time needed for wheel $C$ to reach an angular speed of $100 \text{rev/min}$, assuming the belt does not slip. (Hint: If the belt does not slip, the linear speeds at the two rims must be equal)

**SLN:**

$$\omega_C = \frac{100 \text{rev}}{\text{min}} = (100 \text{rev})(\frac{2\pi}{\text{rev}})(\frac{3.14 \text{rad}}{\pi})(\frac{\text{min}}{60 \text{s}}) = 10.47 \text{rad/s}$$

$$v_A = v_C \Rightarrow r_A \omega_A = r_C \omega_C \Rightarrow \omega_A = \frac{r_C \omega_C}{r_A} = \frac{(0.25)(10.47)}{0.1} = 26.18 \text{rad/s}$$

considering wheel $A$

$$\omega = \omega_0 + \alpha t \Rightarrow 26.18 = 0 + (1.6)t \Rightarrow t = \frac{26.18}{1.6} \Rightarrow t = 16.36 \text{s}$$
Problem 7: Two particles, each with mass $m = 0.85kg$, are fastened to each other, and to rotation axis at $O$ by two uniform thin rods, each with length $d = 5.6cm$ and mass $M = 1.2kg$ as shown in the figure. The combination rotates around the rotation axis with the angular speed $\omega = 0.3 \text{rad/s}$. \{l_{\text{particle}} = mr^2 \text{ and } l_{\text{solid disc}} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2\}

a) Measured about $O$, what is the combination’s rotational inertia?

b) Measured about $O$, what is the combination’s rotational kinetic energy?

SLN:

a) $I = \sum m_ir_i^2$

$I = I_1 + I_2 + I_3 + I_4 = \left\{\frac{1}{4}M\left(\frac{d}{2}\right)^2 + \frac{1}{12}Md^2\right\} + \{md^2\} + \left\{\frac{1}{4}M\left(\frac{3d}{2}\right)^2 + \frac{1}{12}Md^2\right\} + m(2d)^2 = md^2 + 4md^2 + \frac{2Md^2}{4} + \frac{2M9d^2}{16}$

$= 5md^2 + \frac{8Md^2}{3} = (5)(0.85)(0.056)^2 + \frac{8(1.2)(0.056)^2}{3} = 0.023kgm^2$

b) $\Sigma K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.023)(0.3)^2 = 0.001\text{Joule}$
**Problem 8:** A machine part consists of three disks linked by lightweight struts as shown in the Figure.

a) What is this body’s moment of inertia about an axis through the center of disk A, perpendicular to the plane of the diagram?

b) What is this body’s moment of inertia about an axis through the center of disk B and C?

c) What is the body’s kinetic energy if it rotates about the axis through A with angular speed \( \omega = 4 \text{ rad/s} \)?

**SLN:**

a) \( I_A = \sum m_i r_i^2 = m_B r_B^2 + m_C r_C^2 = (0.1)(0.5)^2 + (0.2)(0.4)^2 = 0.057 \text{kgm}^2 \)

b) \( I_{BC} = \sum m_i r_i^2 = m_A r_A^2 = (0.3)(0.4)^2 = 0.048 \text{kgm}^2 \)

c) \( \sum K_{RA} = \frac{1}{2} I_A \omega^2 = \frac{1}{2} (0.057)(4)^2 = 0.456 \text{ Joule} \)

**Problem 9:** A cord connected at one end to a block which can slide on an inclined plane has its other end wrapped around a cylinder resting in a depression at the top of the plane as shown in the Figure. The coefficient of kinetic friction between the block and the surface is \( \mu_k = 0.055 \). Determine the speed of the block after it has travelled 1.8m along the plane, starting from rest. \( \{I_{cylinder} = \frac{1}{2} mr^2\} \)
Problem 10: A 15kg object and a 10kg object are suspended, joined by a cord that passes over a pulley with a radius of 10cm and a mass of 3kg. The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest 3m apart. Treat the pulley as a uniform disc, and determine the speeds of the two objects as they pass each other. \( I_{\text{solid disc}} = \frac{1}{2} mR^2 \)

Problem 11: A spring with 1200 \(N/m\) spring constant is used to launch a solid sphere having a mass of \( m = 4kg \) and a radius of \( R = 0.2m \) up a curved rump. The spring has relaxed length of 0.6m and is compressed to a new length of 0.2m. After being launched the sphere starts rolling without slipping. Determine the maximum height the box will reach on the ramp. \( I_{\text{solid sphere}} = \frac{2}{5} mR^2 \)

Problem 12: A solid sphere having a mass of \( m = 4kg \) and a radius of \( R = 0.2m \) is rolling without slipping. It passes point \( O \) with a linear speed of \( v_o = 4m/s \). Find the maximum height, \( h_{\text{max}} \), the sphere can roll up on the \( 37^\circ \) incline before coming to rest. \( I_{\text{solid sphere}} = \frac{2}{5} mR^2 \)
**Problem 13:** The fishing pole in the Figure makes an angle of $20^\circ$ with the horizontal. What is the torque exerted by the fish about an axis perpendicular to the page and passing through fisher’s hand?

**SLN:**

\[ \tau = \vec{r} \times \vec{F} \]

\[ \vec{r} = 2 \cos 20 \hat{i} + 2 \sin 20 \hat{j} = (1.88 \hat{i} + 0.68 \hat{j}) \text{m} \]

\[ \vec{F} = 100 \cos 323 \hat{i} + 100 \sin 323 \hat{j} = (79.86 \hat{i} - 60.18 \hat{j}) \text{N} \]

\[ \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.88 & 0.68 & 0 \\ 79.86 & -60.18 & 0 \end{vmatrix} = \hat{i} \begin{vmatrix} 0.68 & 0 \\ -60.18 & 79.86 \end{vmatrix} + \hat{j} \begin{vmatrix} 0 & 1.88 \\ 0 & 79.86 \end{vmatrix} + \hat{k} \begin{vmatrix} 1.88 & 0.68 \\ 79.86 & -60.18 \end{vmatrix} = \hat{i}[(0.68)(0) - (0)(-60.18)] + \hat{j}[(0)(79.86) - (1.88)(0)] + \hat{k}[(1.88)(-60.18) - (0.68)(79.86)] = 0 + 0 + \hat{k}(-113.14 - 54.30) = -167.4 \hat{k} \text{Nm} \]

*if only the magnitude of the torque is needed:*

\[ |\vec{\tau}| = |\vec{r}||\vec{F}| \sin \theta = (2)(100) \sin 57 = 167.7 \text{Nm} \]

**Problem 14:** A wheel of radius $R$, mass $M$, and moment of inertia $I$ is mounted on a frictionless horizontal axle, as shown in the Figure. A light cord is wrapped around the wheel supports an object of mass $m$. Calculate the angular acceleration of the wheel, the linear acceleration of the object and the tension in the cord.
Problem 15: The Figure shows a uniform disc, with mass $M = 2.5\, kg$ and radius $R = 20\, cm$, mounted on a fixed horizontal axle. A block with mass $m = 1.2\, kg$ hangs from a massless cord that is wrapped around the rim of the disc. Find the acceleration of the falling block, the angular acceleration of the disc, and the tension in the cord. The cord does not slip, and there is no friction on the axle. $\left\{ I_{solid\ disc} = \frac{1}{2} m R^2 \right\}$

SLN:
The block has a linear motion;
\[
\sum \vec{F} = m\vec{a} \quad \Rightarrow \quad T - F_g = m(-a) \quad \Rightarrow \quad T = mg - ma
\]

The disc has an angular motion;
\[
\sum \vec{r} = I \vec{a} \quad \Rightarrow \quad -TR = I(-\alpha) \quad \Rightarrow \quad TR = I \left( \frac{a}{R} \right) \quad \Rightarrow \quad T = \frac{Ia}{R^2}
\]

\[
T = T \quad \Rightarrow \quad mg - ma = \frac{Ia}{R^2} \quad \Rightarrow \quad mg = \frac{Ia}{R^2} + ma \quad \Rightarrow \quad mg = \left( \frac{\frac{1}{2} MR^2}{R^2} + m \right) a \quad \Rightarrow \quad a = \frac{mg}{\left( \frac{1}{2} M + m \right)} \quad \Rightarrow
\]

\[
a = \frac{(1.2)(9.8)}{(0.5)(2.5) + (1.2)} = 4.8 \, m/s^2
\]

\[
\alpha = \frac{a}{R} = \frac{4.8}{0.2} = 24 \, rad/s^2
\]

\[
\Rightarrow \quad T = mg - ma = (1.2)(9.8) - (1.2)(4.8) = 6N
\]
**Problem 16:** Two blocks are connected by a string of negligible mass passing over a pulley of radius \( r = 0.25m \) and moment of inertia \( I = 3kgm^2 \). The block on the rough table has a mass of \( 8kg \) and the coefficient of kinetic friction between the table and the box is 0.1 whereas the free hanging block has a mass of \( m = 6kg \).

a) Draw the free body diagrams for the objects and the pulley.

b) Find the linear acceleration of the objects, the angular acceleration of the pulley, and the tensions in the cord.

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**Problem 17:** Two blocks having \( m_1 \) and \( m_2 \) are connected to each other by a light cord that passes over two identical frictionless pulleys, each having a moment of inertia \( I \) and radius \( R \), as shown in the figure. Find the linear acceleration of each block and the tensions \( T_1 \), \( T_2 \) and \( T_3 \) in the cord. (Assume no slipping between cord and pulleys.)

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**Problem 18:** In the Figure, block 1 has mass \( m_1 = 460g \), block 2 has mass \( m_2 = 500g \), and the pulley which is mounted on a horizontal axle with negligible friction, has radius \( R = 5cm \). When released from rest, block 2 falls 75cm in 5s without the cord slipping on the pulley.

a) What is the magnitude of the acceleration of the blocks? (\( ans: a = 0.06 \text{m/s}^2 \))

b) What are the tensions \( T_1 \) and \( T_2 \)? (\( ans: T_1 = 4.87N, T_1 = 4.54N \))

c) What is the magnitude of the pulley’s angular acceleration? (\( ans: \alpha = 1.2 \text{rad/s}^2 \))

d) What is the pulley’s rotational inertia? (\( ans: I = 0.0138kgm^2 \))