EEE 461 Communication Systems II

Lecture Presentation 14

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☞ *7.1 (Couch) Error Probabilities for Binary Signaling*

☞ General Results for the Probability of Error

$$s(t) = \begin{cases} s_1(t), & 0 < t \le T, \text{ for a binary } 1 \\ s_2(t), & 0 < t \le T, \text{ for a binary } 0 \end{cases}$$

If $s_1(t) = -s_2(t)$, the signaling is called antipodal.

$$r_0(t) = \begin{cases} r_{01}(t), & 0 < t \le T, \text{ for a binary } 1 \\ r_{02}(t), & 0 < t \le T, \text{ for a binary } 0 \end{cases}$$

t_0 is the sampling time.

$$r_0(t_0) = \begin{cases} r_{01}(t_0), & 0 < t \le T, \text{ for a binary } 1 \\ r_{02}(t_0), & 0 < t \le T, \text{ for a binary } 0 \end{cases}$$

$r_0 = r_0(t_0)$ is a random variable that has a continuous distribution because the channel noise corrupts the signal.

r_0 is also called a test statistic.

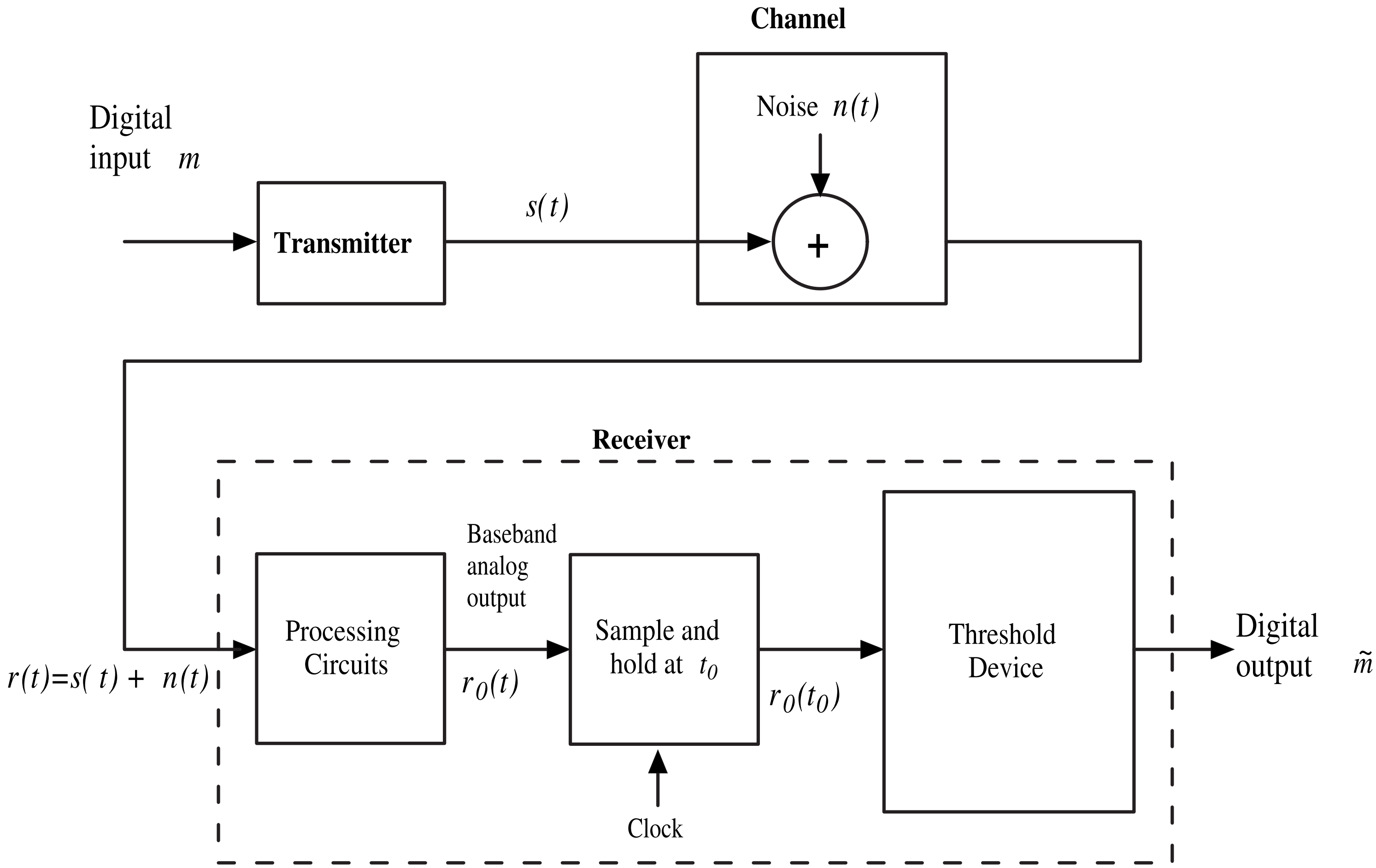

Figure 1: General binary communication system.

- When $r_0 = r_{01}$, the conditional pdf $f(r_0|s_1)$
- When $r_0 = r_{02}$, the conditional pdf $f(r_0|s_2)$

$$P(\text{error}|s_1 \text{ sent}) = \int_{-\infty}^{V_T} f(r_0|s_1) dr_0$$

$$P(\text{error}|s_2 \text{ sent}) = \int_{V_T}^{\infty} f(r_0|s_2) dr_0$$

$$P_e = P(\text{error}|s_1 \text{ sent}) P(s_1 \text{ sent}) + P(\text{error}|s_2 \text{ sent}) P(s_2 \text{ sent})$$

$$P_e = P(s_1 \text{ sent}) \int_{-\infty}^{V_T} f(r_0|s_1) dr_0 + P(s_2 \text{ sent}) \int_{V_T}^{\infty} f(r_0|s_2) dr_0 \qquad (1)$$

For equally likely signals, $P(s_1 \text{ sent}) = P(s_2 \text{ sent}) = \frac{1}{2}$

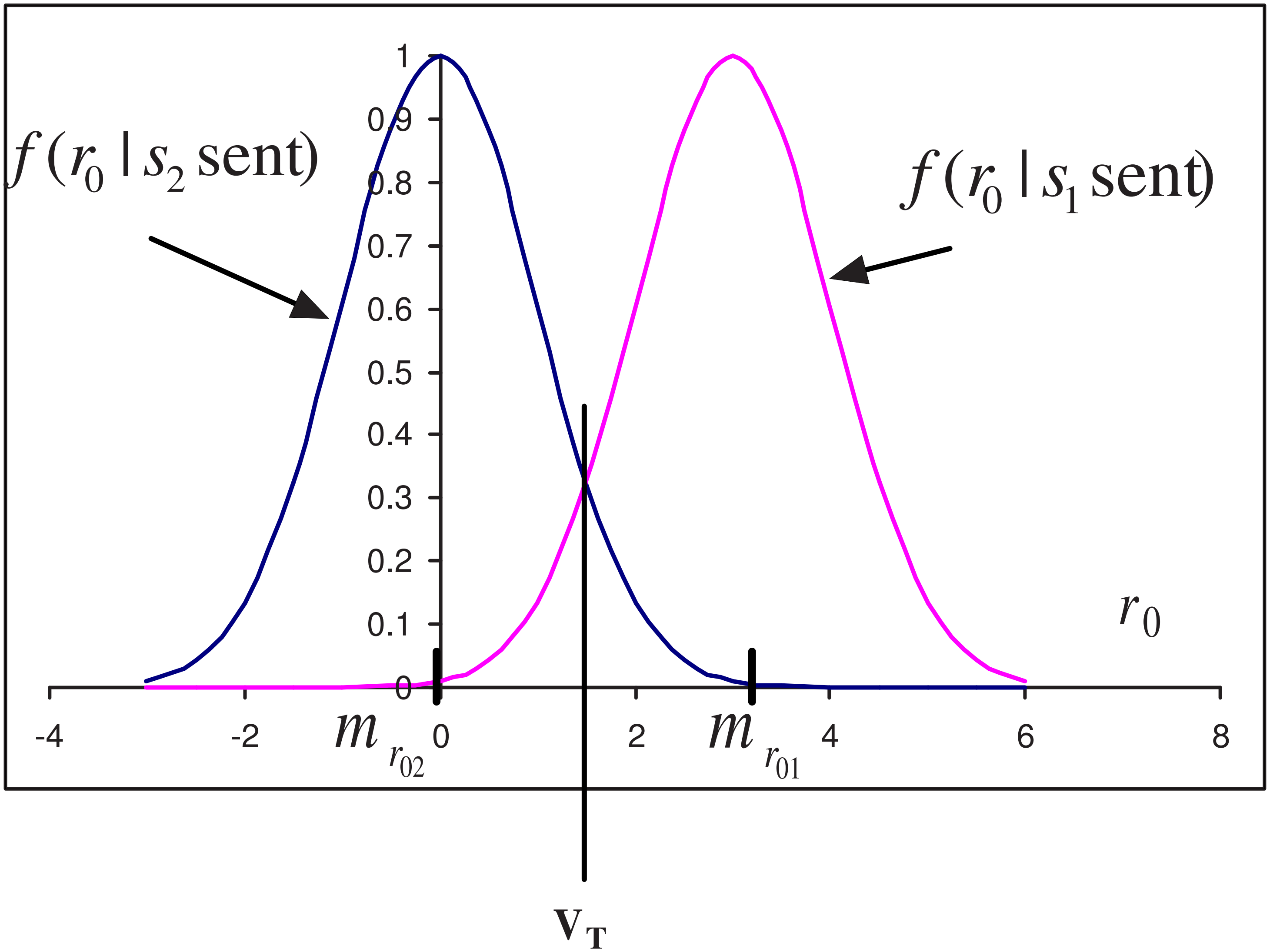

Figure 2: Error probability for binary signaling.

Probability of Error for Gaussian Noise

We assume that the channel noise is zero-mean, WSS, Gaussian random process and that the receiver is linear.

The conditional pdfs resulting from the transmission of binary 1 or a binary 0 are:

$$f(r_0|s_1) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{01})^2/(2\sigma_0^2)}. \tag{2}$$

and

$$f(r_0|s_2) = \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{02})^2/(2\sigma_0^2)}. \tag{3}$$

where $\sigma_0^2 = E[n_0^2] = n_0^2(t_0) = n_0^2(t)$ is the average power of the output noise from the receiver.

Substituting equation (2) and (3) into (1) the P_e becomes

$$P_e = \frac{1}{2} \int_{-\infty}^{V_T} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{01})^2/(2\sigma_0^2)} dr_0 + \frac{1}{2} \int_{V_T}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(r_0 - s_{02})^2/(2\sigma_0^2)}) dr_0 \tag{4}$$

Let $\lambda = (r_0 - s_{02})/\sigma_0$, when we substitute λ into Eq.(4), we get

$$P_e = \frac{1}{2} Q\left(\frac{-V_T + s_{01}}{\sigma_0}\right) + \frac{1}{2} Q\left(\frac{V_T - s_{02}}{\sigma_0}\right) \tag{5}$$

The value of the threshold V_T optimized so that the P_e is minimized.

The optimum threshold becomes,

$$V_T = \frac{s_{01} + s_{02}}{2} \tag{6}$$

By using this optimum threshold in Eq. (5) we get,

$$P_e\big|_{min} = Q\left(\frac{s_{01} - s_{02}}{2\sigma_0}\right) = Q\left(\sqrt{\frac{(s_{01} - s_{02})^2}{4\sigma_0^2}}\right) \tag{7}$$

where it is assumed that $s_{01} > V_T > s_{02}$.