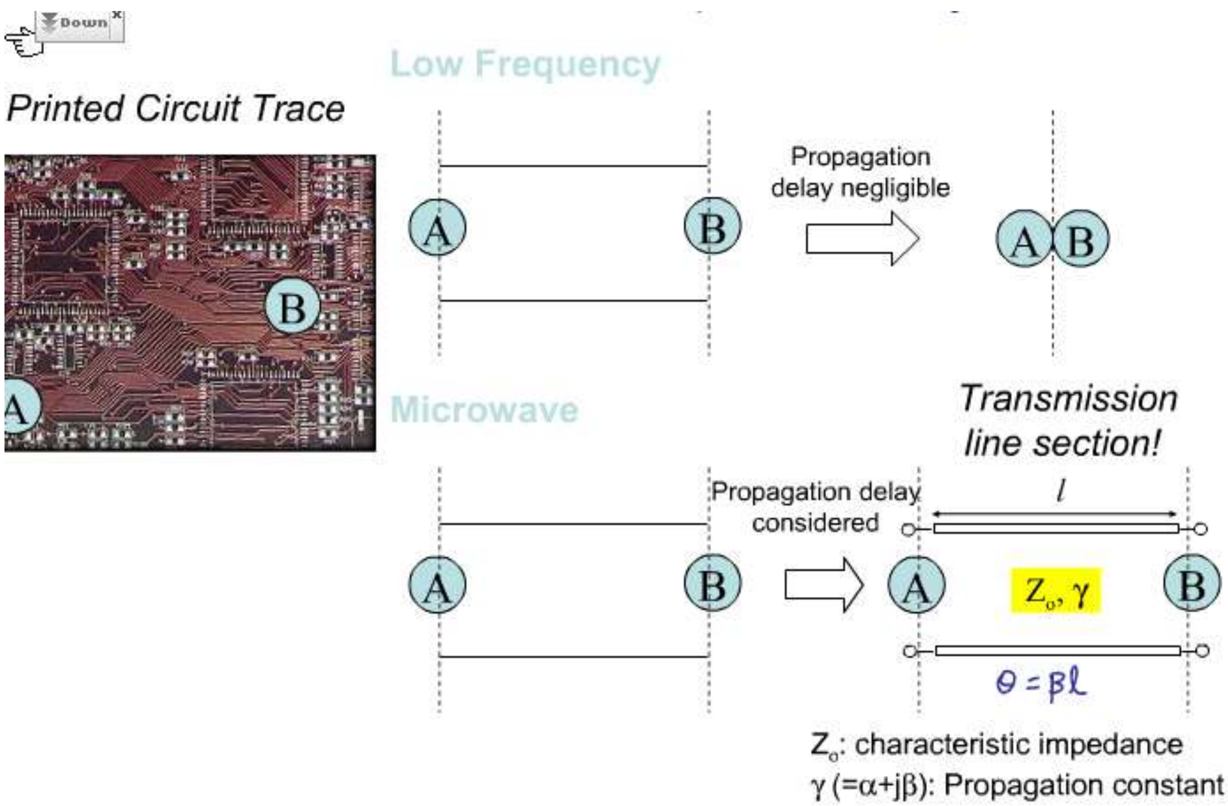
MICROWAVE NETWORK ANALYSIS

At microwave frequencies, equivalent reactive and resistive elements may be connected to form a microwave circuit. In placed of connecting wires, transmission lines and waveguides are used. The lengths, of the connecting links are often several wavelengths and hence propagating effects become very important.

Many of the circuit-analysis techniques and circuit properties that are valid at low frequencies are also valid for microwave circuits. Actually, low-frequency analysis is a special case of microwave circuit analysis.

The variables at low frequencies are voltages and currents.



Equivalent Voltages and Currents

For an arbitrary WG mode with both (+) ly and (-) ly traveling waves, the transverse fields can be written as:

$$\bar{E}_t(x,y,z) = \bar{e}(x,y)\left(A^{+}e^{-j\beta z} + A^{-}e^{j\beta z}\right)$$
$$\bar{H}_t(x,y,z) = \bar{h}(x,y)\left(A^{+}e^{-j\beta z} - A^{-}e^{j\beta z}\right)$$

Let the following equivalent voltage and current waves be introduced:

$$V = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z}$$
$$I = I^{+}e^{-j\beta z} - I^{-}e^{j\beta z}$$

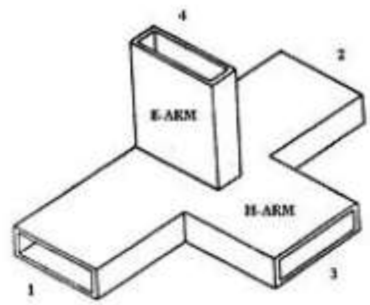
Where,

$$V^{+} = C_1 A^{+} \quad , \quad V^{-} = C_1 A^{-}$$
$$I^{+} = C_2 A^{+} \quad , \quad I^{-} = C_2 A^{-}$$

$C_1 \ and \ C_2$ are the appropriate propagation constants.

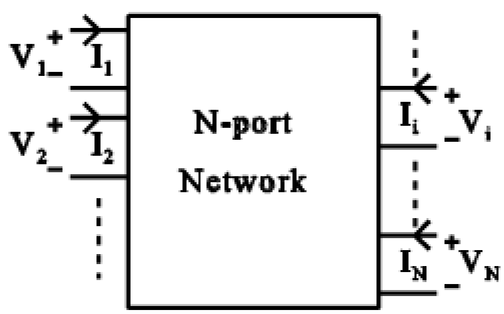
N-PORT CIRCUITS

A WG section is a two port network. i.e. it has one input and one output. Similarly tees and couplers form three port networks.



Magic Tee

Consider the junction of N WG's or TL's (or a combination of the two) that terminate in a common region or junction.



The relationship between the incoming and outgoing waves at various ports of a network describes the system or the network.

We will assume that each guide supports only a single propagating mode.

IMPEDANCE AND ADMITTANCE REPRESENTATION OF N-PORTS

1) Let the terminal planes be chosen sufficiently far from the junction so that the fields on the terminal planes are essentially just those of the incident and the reflected dominant propagating modes.
2) These fields may be defined at the terminal planes in terms of the equivalent currents and voltages.
3) The amplitudes of the incidents waves may be specified and the reflected waves are linear functions of the incident fields. i.e. V_n^- are linear functions of V_n^+.
4) When V_n^+ and V_n^- are known, the corresponding I_n^+ and I_n^- are obtained from the relations:

$$I_n^+ = Y_n V_n^+ \text{ and } I_n^- = Y_n V_n^-$$

For an impedance description, the total currents $I_n = I_n^+ - I_n^-$ at the terminal planes are chosen as independent variables. The N total terminal-plane voltages $V_n = V_n^+ + V_n^-$ are then the dependent variables, and are linearly related to the currents as:

$$\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \ldots & Z_{1N} \\ Z_{21} & Z_{22} & \ldots & Z_{2N} \\ . & . & \ldots & \ldots \\ . & . & \ldots & \ldots \\ . & . & \ldots & \ldots \\ Z_{N1} & Z_{N2} & \ldots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_N \end{bmatrix}$$

The matrix elements Z_{ij} provide a complete description of the electrical properties of N-port circuit.

Properties of the Impedance Matrix:

1) If the junction contains a non-reciprocal medium such as a plasma or a ferrite with an applied dc magnetic biasing field, then in general,

$$Z_{ij} \neq Z_{ji}$$

In other words the impedance matrix is not symmetrical.

2) If the junction does not contain any non-reciprocal media (i.e. if the junction is reciprocal in its electrical properties), then

$$Z_{ij} = Z_{ji}, \ i \neq j$$

And the impedance matrix $[Z]$ is symmetrical.

3) If the junction is lossless (many microwave junction may be approximated as such with negligible error), then all the Z_{ij} must be pure imaginary since there can be no power loss within the junction.

The forgoing discussion applies to the admittance matrix $[Y]$, which relates the currents to the total voltages at the terminal planes.

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot & \cdot & Y_{1N} \\ Y_{21} & Y_{22} & \cdot & \cdot & \cdot & Y_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{N1} & Y_{N2} & \cdot & \cdot & \cdot & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ \cdot \\ V_N \end{bmatrix}$$

Note that Z_{ij} can be found as

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_{k=0}} \qquad for \quad k \neq j$$

In other words, Z_{ij} can be found by driving port j with the current I_j, open circuiting all other ports $\left(I_k = 0,\ for\ k \neq j\right)$, and measuring the open-circuit voltage at port i.

- Z_{ii} is the input impedance seen looking into port i when all other ports are open-circuited.
- Z_{ij} is the transfer impedance between ports i and j when all other ports are open-circuited.

SCATTERING MATRIX FORMULATION (S-PARAMETERS)

Like the impedance or admittance matrix for N-port network, the scattering matrix provides a complete description of the network as seen at its N-ports. It relates the voltage wave incident on the ports to those reflected from ports.

If a wave, with an associated equivalent voltage V_1^+ is incident on the junction at a terminal plane t_1, a reflected wave $S_{11}V_1^+ = V_1^-$ will be produced in line 1, where S_{11} is the reflection coefficient or scattering coefficient, for line 1 with a wave incident on line 1. Waves will also be transmitted, or scattered out of the other junctions and will have amplitudes proportional to V_1^+. These amplitudes can be expressed as:

$$V_n^- = S_{n1}V_1^+, \ n = 2,3,\ldots,N .$$

When waves are incident in all lines, the scattered wave in each line has contributions arising from all the incident waves. Thus:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \cdot \\ \cdot \\ \cdot \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot & \cdot & S_{1N} \\ S_{21} & S_{22} & \cdot & \cdot & \cdot & S_{2N} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{N1} & S_{N2} & \cdot & \cdot & \cdot & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \cdot \\ \cdot \\ \cdot \\ V_N^+ \end{bmatrix}$$

$$\left[V^-\right]=[S]\left[V^+\right]$$

A specific element of $[S]$ can be determined as:

$$S_{ij}=\left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\ \ for\ k\neq j}$$

SCATTERING MATRIX IN TERMS OF THE IMPEDANCE MATRIX

Assume first that the characteristic impedances $Z_{0n}=1$. Then,

$$V_n=V_n^++V_n^- \qquad I_n=I_n^+-I_n^-=V_n^+-V_n^-$$

In matrix form:

$$[V]=\left[V^+\right]+\left[V^-\right] \qquad [I]=\left[V^+\right]-\left[V^-\right]$$

If we multiply each side of the current equation by $[Z]$

$$[Z][I]=[Z]\left[V^+\right]-[Z]\left[V^-\right]$$

Since, $$[Z][I]=[V]=\left[V^+\right]+\left[V^-\right]$$

We get:

$$\left[V^+\right]+\left[V^-\right]=[Z]\left[V^+\right]+[Z]\left[V^-\right]$$

Or,

$$([Z]+[U])[V^-]=([Z]-[U])[V^+]$$

PROPERTIES OF THE SCATTERING MATRIX

For a reciprocal junction, $[S]$, matrix is symmetrical that is $S_{mn}=S_{nm}$, provided the voltages have been chosen so that the power is given by $\frac{|V_n^+|^2}{2}$ for all modes (the characteristic impedance of all the line are the same or unity) .

Symmetry property is expressed as

$[S]^t=[S]$, t denotes transpose.

Unitary Property

For a lossless junction, the total power leaving the N ports must be equal to the total incident power. The mathematical statement of this power-conservation condition is:

$$\sum_{n=1}^{N}|V_n^-|^2=\sum_{n=1}^{N}|V_n^+|^2$$

After some manipulations, this condition can be shown to be equivalent to:

$$[S]^* = \left\{[S]^t\right\}^{-1}$$

A matrix satisfying this condition is called a unitary matrix.

This condition can also be written as:

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij} = \begin{cases} 1 & if \quad i = j \\ 0 & if \quad i \neq j \end{cases}$$

This means that:

i) Any column multiplied with its complex conjugate results in 1.
ii) Any column multiplied with the complex conjugate of any other column results in zero.

THE TRANSMISSION (ABCD) MATRIX

When a number of microwave circuits are connected together in cascade, it is more convenient to represent each in cascade; it is more convenient to represent each junction or circuit by transmission matrix or ABCD matrix. The ABCD matrix of the cascade connection of two or more two-port networks can be easily found by multiplying the ABCD matrices of the individual two ports.