Questions:

1. A spring has a spring constant \( k = 440 \text{N/m} \). How much should the spring be stretched (or compressed) to store 25 J of elastic potential energy? (4P)

Solution:

\[
U = \frac{1}{2} k x^2 \implies x = \pm \sqrt{\frac{2U}{k}} = \pm \sqrt{\frac{2 \cdot 25 \text{J}}{440 \text{N/m}}} = \pm 0.34 \text{m}
\]

2. A person pulls from rest a 50 kg block 40 m along a horizontal frictionless floor by a constant force \( \vec{F}_{\text{app}} = 100 \text{N} \) at an angle \( \theta = 37^\circ \) with the horizontal, as shown in the figure below. (5P)

(a) What is the work done by the force \( \vec{F}_{\text{app}} \) on the block?

Solution:

\[
W = \vec{F}_{\text{app}} \cdot \Delta \vec{r} = F_{\text{app}} \Delta r \cos \theta = 100 \text{N} \cdot 40 \text{m} \cos 37^\circ = 3195 \text{J}
\]
(b) Using the Work-Kinetic Energy theorem, determine the velocity of the block after it moves 40 m.

**Solution:**

\[ W = \Delta K = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 3195}{50}} = 11.3 \frac{m}{s} \]

3. A 1200 kg car has a horizontal speed \( v = 18 \frac{m}{s} \), when it hits a horizontal spring and comes to rest in a distance of 2.2 m. Find the spring constant. (4P)

**Solution:**

The mechanical energy is conserved:

\[ \Delta E_{mech} = (K_f + U_f) - (K_i + U_i) = 0 \]

\[ \left( 0 + \frac{1}{2}kx^2 \right) - \left( \frac{1}{2}mv^2 + 0 \right) = 0 \]

\[ k = \frac{mv^2}{x^2} = \frac{1200 \text{kg} \left( 18 \frac{m}{s} \right)^2}{(2.2m)^2} = 80330 \frac{N}{m} = 80 \text{kN/m} \]

4. A 28g bullet travelling at 230 m/s hits a 3.6 kg block of wood and remains in it. The block+bullet swings up to a maximum height \( h_{max} \) as shown in the figure below. Find \( h_{max} \). (5P)

**Solution:**

The solution to this problem is determined in two steps. First we consider the perfectly inelastic collision of the bullet with the block of wood, then we consider the block with the embedded bullet swinging up to the height \( h_{max} \), where the block will stand still before it swings back.

Collision of bullet with block of wood is perfectly inelastic:

\[ m_Bv_{B_i} + m_Wv_{W_i} = (m_B + m_W)v \]

with \( v_{W_i} = 0 \), we get

\[ v = \frac{m_B}{m_B + m_W}v_{B_i} \]

Now the block+bullet has the velocity \( v \), and the potential energy 0. The block+bullet-Earth system is isolated, therefore the mechanical energy is conserved

\[ \Delta E_{mech} = \frac{1}{2}(m_B + m_W)v^2 - (m_B + m_W)gh_{max} = 0 \implies h_{max} = \frac{v^2}{2g} = \left( \frac{m_B}{m_B + m_W} \right)^2 \frac{v_{B_i}^2}{2g} \]

\[ h_{max} = \left( \frac{0.028 \text{kg}}{3.6 \text{kg} + 0.028 \text{kg}} \right)^2 \frac{(230 \text{m/s})^2}{2 \cdot 9.80 \text{m/s}^2} = 0.16m \]
5. A block of mass \( m = 5 \text{ kg} \) is released from rest at point A and slides on the frictionless track shown in below. (5P)

Determine,

(a) the block’s speed at points B and C, and

**Solution:**

The Block-Earth system is an isolated system without friction \( \implies \) Conservation of mechanical energy:

\[
\Delta E_{mech} = (K_f + U_f) - (K_i + U_i) = 0
\]

\[
\left( mgh_f + \frac{1}{2}mv_f^2 \right) - (mgh_i - 0) = 0
\]

\[
v_f = \sqrt{2g(h_i - h_f)}
\]

so the speed at the points B and C are:

\[
v_B = \sqrt{2 \cdot 9.80 \text{m/s}^2 (5\text{m} - 3.2\text{m})} = 5.94 \text{m/s}
\]

\[
v_C = \sqrt{2 \cdot 9.80 \text{m/s}^2 (5\text{m} - 2\text{m})} = 7.67 \text{m/s}
\]

(b) the net work done by the gravitational force on the block as it moves from point A to point C.

**Solution:**

\[
W_{A\rightarrow C} = \Delta K_{A\rightarrow C} = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2} 5.0 \text{kg} \cdot (7.67 \text{m/s})^2 = 147 \text{J}
\]