STEADY CURRENTS

Stationary charges produce electric fields that are constant in time; hence the term electrostatics is used.

Steady currents produce magnetic fields that are constant in time; hence the term magnetostatics is used.

By steady current we mean a flow of charge which has been going on forever, never increasing, never decreasing.

Current Distribution

If the charges are in motion, they constitute current distributions.

Current Density

Consider a tube of charge with density $\rho_v$. The charges are moving with a mean velocity $\bar{u}$ through a surface $\Delta S$ whose surface normal is $\hat{n}$. 
Over a period \( \Delta t \), the charges move a distance

\[
\Delta \vec{l} = \vec{u} \Delta t
\]

The amount of charge that crosses the tube’s cross-section \( \Delta S \) in \( \Delta t \) is:

\[
\Delta q = \rho_v \Delta v = \rho_v \vec{u} \Delta t \cdot \hat{n} \Delta S
\]

The corresponding current:

\[
\Delta I = \frac{\Delta q}{\Delta t} = \rho_v \vec{u} \cdot \hat{n} \Delta S
\]

Let,

\[
\bar{J} = \rho_v \vec{u} \ (A/m^2), \text{ the current density.}
\]

The total current through a surface \( S \) is:

\[
I = \int_S \bar{J} \cdot \hat{n} \, ds \ (A)
\]

If the surface \( S \) is a closed surface:

\[
I = \oint_S \bar{J} \cdot \hat{n} \, ds \ (A)
\]

For steady currents:

\[
\oint_S \bar{J} \cdot \hat{n} \, ds = 0
\]
This means that the net current through a closed surface is zero. The amount of current entering into $S$ is exactly equal to the amount leaving it. So, steady currents establish a solenoidal vector field. i.e.

$$\nabla \cdot \vec{J} = 0$$

There are two types of electric currents caused by the motion of the free charges:

1) Convection Currents
   These currents are due to the motion of the positively and negatively charged particles in a vacuum or gas.

2) Conduction Currents
   Conduction currents in conductors or semiconductors are caused by drift motion of conduction electrons or holes. They obey the *Ohm’s Law.*
Here, we will concentrate on conduction currents.

**Conduction Currents**

The drift velocity $\bar{u}_e$ of electrons in a conducting material is related to the externally applied electric field $\bar{E}$,

$$\bar{u}_e = -\mu_e \bar{E}$$

Where $\mu_e$ is the electron mobility (material property) in $(m^2/V.s)$.

In a semiconductor, the current flow is due to the motion of both electrons and holes (+ve charges).

$$\bar{u}_h = \mu_h \bar{E}$$

$\mu_h$ is the hole mobility

Since,

$$\bar{J} = \rho_v \bar{u} = \rho_{v_e} \bar{u}_e + \rho_{v_h} \bar{u}_h$$

$$\bar{J} = \left(-\rho_{v_e} \mu_e + \rho_{v_h} \mu_h\right) \bar{E}$$
Let $\sigma = -\rho_v \mu_e + \rho_v \mu_h$ the conductivity of the medium in Simens/meter ($S/m$) or mohos /meter ($\Omega/m$).

In either case,

$$\overline{J} = \sigma \overline{E} \ (A/m^2)$$

This is the Field Theory Expression of the Ohms Law.

Note that, in a perfect dielectric with $\sigma = 0$, $\overline{J} = 0$ and in a perfect conductor $\sigma = \infty$, $\overline{E} = \frac{\overline{J}}{\infty} = 0$.

$\sigma$ is of order of $10^6$ or $10^7 (S/m)$ for a good conductor.

For semiconductors, i.e. for pure Germanium $\sigma = 2.2 \ (S/m)$.

For insulators, the conductivity varies from $10^{-10}$ to $10^{-17} \ (S/m)$. 
Ohm’s Law for a Linear, Homogeneous Conductor of Length $\ell$

When a dc voltage $V$ is applied between the ends of a conductor which has conductivity $\sigma$. In this case the electric field is not zero because the conductor is not isolated but is wired to a source of electromotive source which compiles the charges to move and prevents the equilibrium to be established. Thus electric field exists to sustain the current flow. As electrons move, they encounter some forces called resistance. Based on the Ohm’s Law, we will derive the resistance of the conducting
medium of uniform cross-section of area $S$ and length $\ell$.

The applied electric field is uniform and its magnitude is:

$$ E = \frac{V}{\ell} $$

Since the conductor has uniform cross-section, the magnitude of the current density is:

$$ J = \frac{I}{S} \quad \text{(since } I = \int_{s} J \cdot ds \text{)} $$

Both, the electric field intensity and the current density are in the direction of the current flow in the conductor.

$$ \bar{J} = \sigma \bar{E} $$

Substituting,

$$ \frac{I}{S} = \sigma \frac{V}{\ell} $$

Define the resistance of the conductor,

$$ R = \frac{V}{\frac{I}{\ell}} = \frac{\ell}{\sigma S} \quad (\Omega) $$
POWER DISSIPATION AND JOULE’S LAW

As the free electrons drift inside a conductor there will be collisions between them and the atoms. The kinetic energy of the electron is converted into the thermal energy of the atom in a collision. Following this, the temperature of the conductor raises as the current flows. This heat increase is called the Joule’s heat. It is clear that to maintain an electric current inside a conductor, energy is to be transferred to the electrons continuously. This is dissipated as heat in the conductor.

This dissipated power in a volume $V$ of the conductor can be calculated as:

$$ P = \int_{V} \vec{J} \cdot \vec{E} \, dv $$  Watts.

For a linear conductor:

$$ P = \int_{V} \sigma \vec{E} \cdot \vec{E} \, dv = \int_{V} \sigma |\vec{E}|^2 \, dv $$