PHYS101 Interm Exam - Solution Set

Department of Physics

Spring 2014 - May 15, 2014

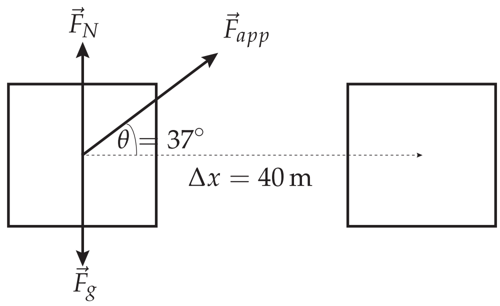
Questions:

1. A spring has a spring constant $k = 440N/m$. How much should the spring be stretched (or compressed) to store 25 J of elastic potential Energy? (4P)

   Solution:

$$U = \frac{1}{2}kx^2 \Longrightarrow x = \pm\sqrt{\frac{2U}{k}} = \pm\sqrt{\frac{2 \cdot 25J}{440N/m}} = \pm 0.34m$$

2. A person pulls from rest a 50 kg block 40 m along a horizontal frictionless floor by a constant force $\vec{F}_{app} = 100\text{N}$ at an angle $\theta = 37°$ with the horizontal, as shown in the figure below. (5P)



   (a) What is the work done by the force $\vec{F}_{app}$ on the block?
   Solution:

$$W = \vec{F}_{app} \cdot \Delta\vec{r} = F_{app}\Delta r \cos\theta = 100N\,40m\,\cos 37° = 3195J$$

(b) Using the Work-Kinetic Energy theorem, determine the velocity of the block after it moves 40 m.

Solution:

$$W = \Delta K = \frac{1}{2}mv^2 \Longrightarrow v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \cdot 3195J}{50kg}} = 11.3\frac{m}{s}$$

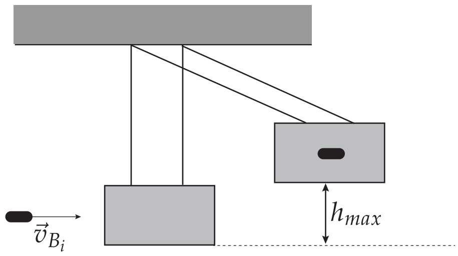
3. A 1200 kg car has a horizontal speed $v = 18m/s$, when it hits a horizontal spring and comes to rest in a distance of 2.2 m. Find the spring constant. (4P)

Solution:

The mechanical energy is conserved:

$$\begin{aligned} \Delta E_{mech} &= (K_f + U_f) - (K_i + U_i) = 0 \\ & \left(0 + \frac{1}{2}kx^2\right) - \left(\frac{1}{2}mv^2 + 0\right) = 0 \\ k &= \frac{mv^2}{x^2} = \frac{1200kg\,(18m/s)^2}{(2.2m)^2} = 80330\frac{N}{m} = 80\frac{kN}{m} \end{aligned}$$

4. A 28g bullet travelling at 230 m/s hits a 3.6 kg block of wood and remains in it. The block+bullet swings up to a maximum height h_{max} as shown in the figure below. Find h_{max}. (5P)



Solution:

The solution to this problem is determined in two steps. First we consider the perfectly inelastic collision of the bullet with the block of wood, then we consider the block with the embedded bullet swinging up to the height h_{max}, where the block will stand still before it swings back.

Collision of bullet with block of wood is perfectly inelastic:

$$m_B v_{B_i} + m_W v_{W_i} = (m_B + m_W)v,$$

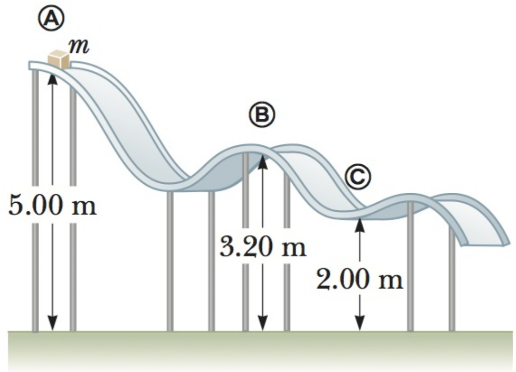
with $v_{W_i} = 0$, we get

$$v = \frac{m_B}{m_B + m_W} v_{B_i}.$$

Now the block+bullet has the velocity v, and the potential energy 0. The block+bullet-Earth system is isolated, therefore the mechanical energy is conserved

$$\begin{aligned} \Delta E_{mech} &= \frac{1}{2}(m_B + m_W)v^2 - (m_B + m_W)gh_{max} = 0 \Longrightarrow h_{max.} = \frac{v^2}{2g} = \left(\frac{m_B}{m_B + m_W}\right)^2 \frac{v_{B_i}^2}{2g} \\ h_{max.} &= \left(\frac{0.028kg}{3.6kg + 0.028kg}\right)^2 \frac{(230m/s)^2}{2 \cdot 9.80m/s^2} = 0.16m \end{aligned}$$

5. A block of mass m = 5 kg is released from rest at point A and slides on the frictionless track shown in below. (5P)



Determine,

(a) the block's speed at points B and C, and

Solution:

The Block-Earth system is an isolated system without friction $\Longrightarrow$ Conservation of mechanical energy:

$$\begin{aligned} \Delta E_{mech} &= (K_f + U_f) - (K_i + U_i) = 0 \\ \left(mgh_f + \frac{1}{2}mv_f^2\right) - (mgh_i - 0) &= 0 \\ v_f &= \sqrt{2g(h_i - h_f)} \end{aligned}$$

so the speed at the points B and C are:

$$\begin{aligned} v_B &= \sqrt{2 \cdot 9.80m/s^2(5m - 3.2m)} = 5.94\frac{m}{s} \\ v_C &= \sqrt{2 \cdot 9.80m/s^2(5m - 2m)} = 7.67\frac{m}{s} \end{aligned}$$

(b) the net work done by the gravitational force on the block as it moves from point A to point C.

Solution:

$$W_{g_{A\to C}} = \Delta K_{A\to C} = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2}\,5.0kg\,(7.67m/s)^2 = 147J$$