INTEGRALS CONTAINING VECTOR FUNCTIONS

In electromagnetic work some integrals need to be evaluated which contain vector functions. i. e.

$$\int_v \bar{F}\, dv, \quad \int_c V\, d\bar{l}, \quad \int_c \bar{F}.d\bar{l}, \quad \int_s \bar{A}.d\bar{s}$$

1) Volume Integral:

$$\int_v \bar{F}\, dv = \iiint \bar{F} dv \Rightarrow$$ The result is a vector

2) Line Integral:

$$\int_c V d\bar{l} \Rightarrow$$ The result is a vector

3) Line Integral

a) $\int_c \bar{F}.d\bar{l} \Rightarrow$ The result is a scalar.

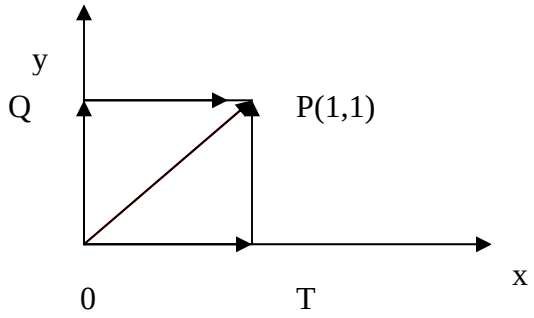
b) $\oint_c \bar{F}.d\bar{l} \Rightarrow$ The result is a scalar.

4) Surface Integral

a) $\int_s \bar{A}.d\bar{s} \Rightarrow$ The result is a scalar.

b) $\oint_s \bar{A}.d\bar{s} \Rightarrow$ The result is a scalar.

Example: Evaluate $\int_0^P \rho^2 d\bar{\rho}$, for $\rho^2 = x^2 + y^2$ and



Along OP:

$$\int_0^P \rho^2 d\bar{l} = \hat{a}_\rho \int_0^{\sqrt{2}} \rho^2 d\rho = \hat{a}_\rho \frac{\rho^3}{3}\Big|_0^{\sqrt{2}} = \hat{a}_\rho \frac{2\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}\left(\hat{a}_x \cos\left(\frac{\pi}{4}\right) + \hat{a}_y \sin\left(\frac{\pi}{4}\right)\right) = \frac{2}{3}\left(\hat{a}_x + \hat{a}_y\right)$$

Along OQP:

$$\int_o^P \left(x^2 + y^2\right) d\bar{l} = \hat{a}_y \int_0^Q y^2 dy + \hat{a}_x \int_Q^P \left(1 + x^2\right) dx = \frac{1}{3}\left(4\hat{a}_x + \hat{a}_y\right)$$

Along OTP:

$$\hat{a}_x \int_0^1 x^2 dx + \hat{a}_y \int_0^1 \left(1 + y^2\right) dy = \frac{1}{3}\left(\hat{a}_x + 4\hat{a}_y\right)$$