DEL OPERATOR

Many of the quantities we deal in electromagnetics are vectors and both their magnitudes and directions may vary with spatial position. In vector calculus, we use three fundamental operators to describe the spatial variation of vectors and scalars; gradient, divergence and curl.

Del operator (∇) is the vector differential operator used in spatial variation.

1) The gradient of a scalar, $\nabla V \rightarrow vector$.
2) The divergence of a vector, $\nabla . \bar{A} \rightarrow scalar$.
3) The curl of a vector, $\nabla X \bar{A} \rightarrow vector$.
4) The Laplacian of a scalar, combination of a divergence with gradient, $\nabla^2 V \rightarrow scalar$

In Cartesian Coordinates:

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

In Cylindrical Coordinates:

$$\nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{\partial}{\rho \partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z$$

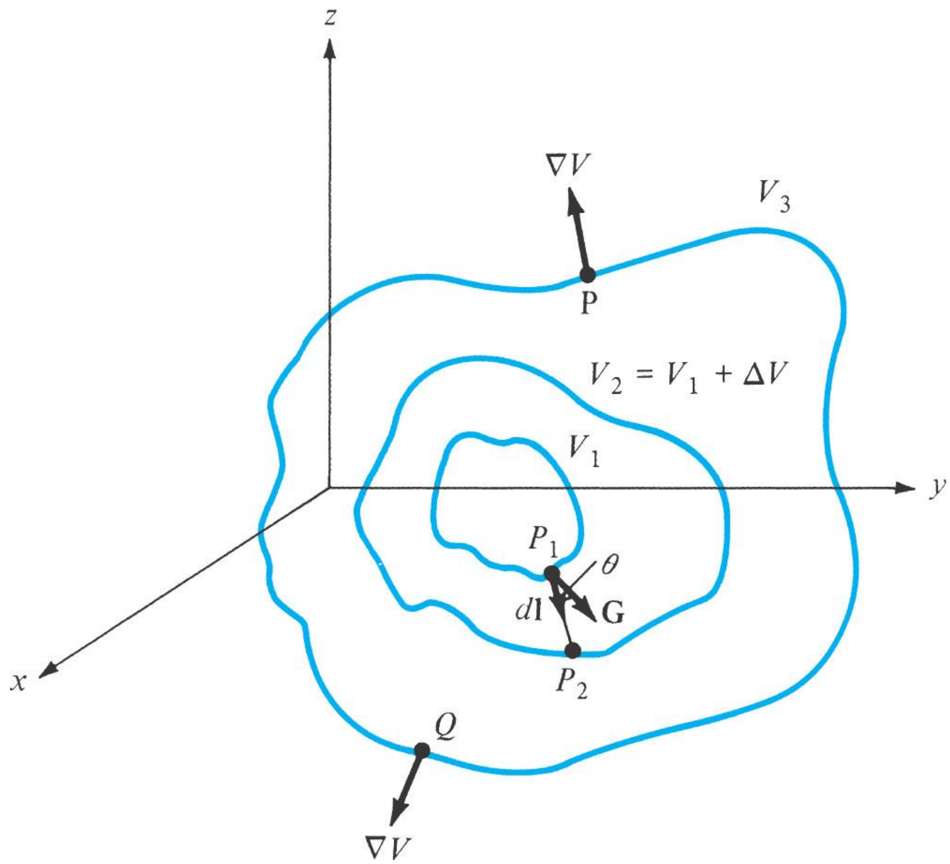
In Spherical Coordinates:

$$\nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{\partial}{r \partial \theta}\hat{a}_\theta + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

GRADIENT OF A SCALAR

The gradient of a scalar field *V*, is a vector that represents both the magnitude and direction of the maximum space rate of increase of *V*. i.e the electrical potential in a region of space.

To obtain the mathematical expression of the gradient, evaluate the difference in the field dV between points P_1 and P_2 where V_1 , V_2 and V_3 are contours on which V is constant.



Suppose that $V_1(x, y, z)$ is the potential at point P_1 and $V_2(x+dx, y+dy, z+dz)$ is the potential at a nearby point P_2. dx, dy, and dz are components of the differential length $d\bar{l}$.

$$d\bar{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

From calculus:

$dV = V_2 - V_1$ is given by:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$dV = \left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right).\left(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z\right)$$

Let,

$$\bar{G} = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Then,

$$dV = \bar{G}.d\bar{l} = G\cos\theta dl$$

$$\frac{dV}{dl} = G\cos\theta$$

Where, $d\bar{l}$ is the displacement from P_1 to P_2 and θ is the angle between $\bar{G}$ and $d\bar{l}$. We notice that $\frac{dV}{dl}$ is max when $\theta = 0$, that is when $d\bar{l}$ is in the direction of $\bar{G}$. Hence,

$$\left.\frac{dV}{dl}\right|_{\max} = \frac{dV}{dn} = \left|\bar{G}\right|$$

$\frac{dV}{dn}$, is the normal derivative. Thus $\bar{G}$ has its magnitude and direction as those of the maximum rate of *V*. $\bar{G}$ is the gradient of *V*

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Note the following properties of the gradient:

1) ∇V at any point is $\perp$ to the constant *V* surface that passes through that point.

2) The projection of ∇V in the direction of a unit vector $\hat{a}_l$ is $\nabla V.\hat{a}_l$ and it is called directional derivative of *V* along $\hat{a}_l$.