CURL OF A VERTOR FIELD

The net outward flux of a vector field $\bar{A}$ through a surface bounding a volume indicates the presence of a source. This source may be called a flow source and the divergence of $\bar{A}$ is a measure of the strength of the flow (scalar) source.

There is another kind of source called vortex (vector) source which causes a circulation of a vector field around it.

The circulation of vector field $\bar{A}$ around a closed path C is:

Circulation of $\bar{A} = \oint_C \bar{A}.d\bar{l}$

The curl of $\bar{A}$, is a rotational vector whose magnitude is the maximum circulation of $\bar{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make circulation maximum. That is:

$$\text{Curl } \bar{A} = \nabla X \bar{A} = Lim_{\Delta S \to 0} \left(\frac{\oint_C \bar{A}.d\bar{l}}{\Delta S} \right)_{\max} \hat{a}_n$$

∇S is the surface bounded by C and $\hat{a}_n$ is the unit vector normal to ΔS. $\hat{a}_n$ is determined by the right hand rule.

$$\left(curl\ A\right)_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

$$\left(curl\ A\right)_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}$$

$$\left(curl\ A\right)_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

In Cartesian Coordinates:

$$\nabla X\bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

In Cylindrical Coordinates:

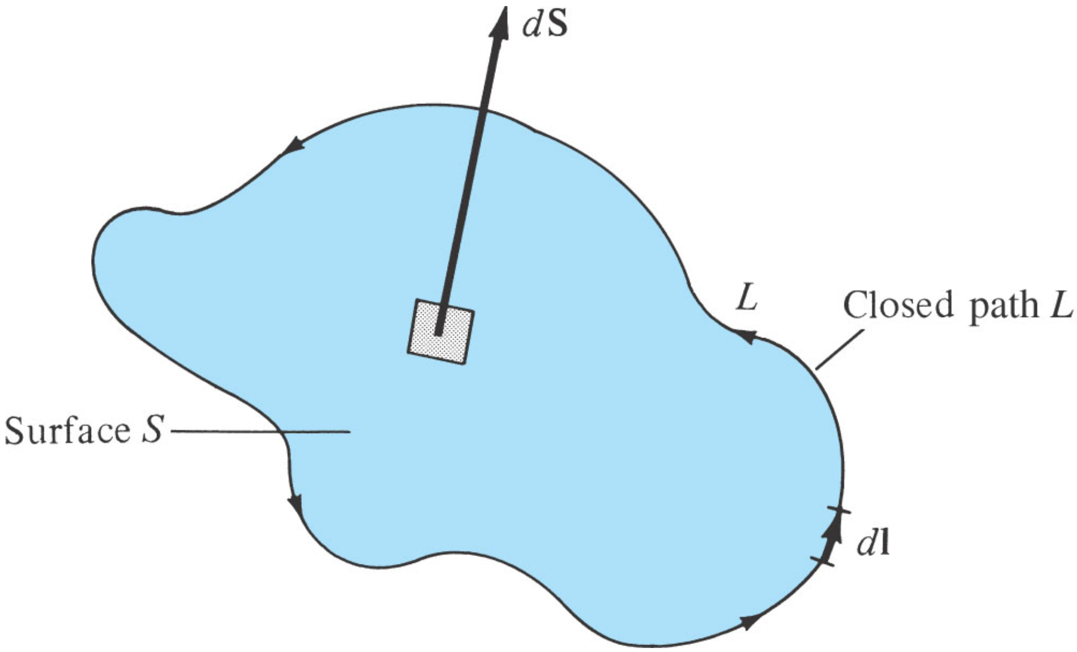
$$\nabla X\bar{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

In Spherical Coordinates:

$$\nabla X\bar{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

STOKES'S THEOREM

Consider an open surface *S* which is bounded by a closed path *C*. The vector field $\bar{A}$ is defined everywhere on *S* and on *C*.



$$\oint_C \bar{A}.d\bar{l} = \int_S \nabla X \bar{A}.d\bar{s}$$

Stokes's Theorem states that the circulation of a vector field $\bar{A}$ around a closed path is equal to the surface integral of the curl of $\bar{A}$ over the open surface *S* bounded by *C*.

If $\nabla X\overline{A} = 0$, vector field $\overline{A}$ is said to be irrotational or conservative field.

If $\nabla X\overline{A} = 0$, for a scalar V , $\nabla X\left[\nabla V\right] = 0$, since curl of a gradient is zero.

If $\nabla X\overline{A} = 0$, then $\oint_C \overline{A}.d\overline{l} = 0$.

HELMHOLTZ'S THEOREM

A vector field is determined within an additive constant if both its divergence and its curl are specidied.