Faculty of Engineering

DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING

EENG (INFE)115 Introduction to Logic Design

Instructors:

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Midterm EXAMINATION

November 24, 2015

Duration : 100 minutes

Number of Questions: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	SOLUTIONS
SURNAME	

Question		**Points**
1		25
2		25
3		25
4		25
TOTAL		100

Read the following instructions carefully:

1. **Calculators** are not allowed.
2. Switch off **mobile phones** and **do not borrow** any stationery from your friends.
3. In your solutions, **show all details** you claim credit for.

Question 1

a) Convert decimal 0.8125 to binary. (**3 pts**.)

	Integer		Fraction	Coefficient
0.8125 x 2 =	1	+	0.6250	$a_{-1} = 1$
0.6250 x 2 =	1	+	0.250	$a_{-2} = 1$
0.250 x 2 =	0	+	0.500	$a_{-3} = 0$
0.500 x 2 =	1	+	0.0000	$a_{-4} = 1$

$(0.8125)_{10} = (0.1101)_2$

b) Convert the hexadecimal number C1A9 to binary, and to octal. (**4 pts**.)

C	1	A	9
1100	0001	1010	1001

001	100	000	110	101	001
1	4	0	6	5	1

$(C1A9)_{16} = (1100\ 0001\ 1010\ 1001)_2 = (140651)_8$

c) Convert the following binary number to hexadecimal and to octal: 10.010 (**4 pts**.)

$(10.010)_2 = (2.4)_{16} = (2.2)_8$

d) Convert decimal +37 and +58 to binary, using the signed-2's-complement representation and enough digits to accommodate the numbers. Then perform the binary equivalent of (+37) + (-58), (-37) + (+58), and (-37) + (-58). (**10 pts**.)

Integer	Remainder
37	
18	1
9	0
4	1
2	0
37	0
0	1

$(37)_{10} = (100101)_2$

Integer	Remainder
58	
29	0
14	1
7	0
3	1
1	1
0	1

$(58)_{10} = (111010)_2$

The summation of 37 and 58 is 95 and needs 7 bits. 1 extra bit is needed for the sign of the number. Therefore we require 8 bits to accommodate the each number.

$(37)_{10} = (00100101)_2$ $(-37) = 11011011$

$(-58) = 11000110$

$(58)_{10} = (00111010)_2$

(+37) + (-58) = -21

	0	0	1	0	0	1	0	1
+	1	1	0	0	0	1	1	0
	1	1	1	0	1	0	1	1

No carry. This means that the result is negative. In order to find the result put a minus sign and take the 2's complement of the result.

$-(00010101)_2 = (-21)_{10}$

(-37) + (+58)=21

	1	1	0	1	1	0	1	1
+	0	0	1	1	1	0	1	0
~~1~~	0	0	0	1	0	1	0	1

$(00010101)_2 = (21)_{10}$

(-37) + (-58)=

	1	1	0	1	1	0	1	1
+	1	1	0	0	0	1	1	0
~~1~~	1	0	1	0	0	0	0	1

Since the most significant bit is 1 the result is negative. $-(01011111)_2 = (-95)_{10}$

e) Decode the following ASCII code if the most significant bit is parity bit and determine the parity used: odd or even (**4 pts**.)

$$\underbrace{11000101}_{E}\underbrace{00101110}_{.}\underbrace{01001101}_{M}\underbrace{00101110}_{.}\underbrace{01010101}_{U}$$

Since we have even number of 1 in each 8 bits, EVEN parity is used.

American Standard Code for Information Interchange (ASCII)

	$b_7b_6b_5$							
$b_4b_3b_2b_1$	**000**	**001**	**010**	**011**	**100**	**101**	**110**	**111**
0000	NUL	DLE	SP	0	@	P	`	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	"	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	'	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	l	|
1101	CR	GS	–	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	–	o	DEL

Question 2

a) Given the Boolean expression F = x'y + xyz': (**7 pts**.)

i. Derive an algebraic expression for the complement F'.

ii. Show that F·F' = 0.

$$F' = (x + y')(x' + y' + z)$$

$$F \cdot F' = (x'y + xyz')(x + y')(x' + y' + z) = \left(\underbrace{x'yx}_{0} + \underbrace{x'yy'}_{0} + \underbrace{xxyz'}_{xyz'} + \underbrace{xyz'y'}_{0} \right)(x' + y' + z)$$

$$F \cdot F' = \underbrace{xyz'x'}_{0} + \underbrace{xyz'y'}_{0} + \underbrace{xyz'z}_{0} = 0$$

b) Express the following function as a sum of minterms and as a product of maxterms (**8 pts**.)

$$F(w,x,y,z) = x'(z + wy) + wy(x' + z') + w'y$$

$$F(w,x,y,z) = x'z + wx'y + wx'y + wyz' + w'y = x'z + wx'y + wyz' + w'y$$

$$F(w,x,y,z) = (w + w')x'(y + y')z + wx'y(z + z') + w(x + x')yz' + w'(x + x')y(z + z')$$

$$F(w,x,y,z) = wx'yz + wx'y'z + w'x'yz + w'x'y'z + wx'yz + wx'yz'$$
$$+ wxyz' + wx'yz' + w'xyz + w'xyz' + w'x'yz + w'x'yz'$$

$$F(w,x,y,z) = m_{11} + m_9 + m_3 + m_1 + m_{11} + m_{10} + m_{14} + m_{10} + m_7 + m_6 + m_3 + m_2$$

$$F(w,x,y,z) = \sum(1,2,3,6,7,9,10,11,14)$$

$$F(w,x,y,z) = \prod(0,4,5,8,12,13,15)$$

c) Given the Boolean function F = xy'z + x'y'z + xyz (**10 pts.**)
  i. List the truth table
  ii. Simplify the function using **Boolean algebra**
  iii. List the truth table of the simplified function

x	y	z	x'	y'	xy'z	x'y'z	xyz	xy'z + x'y'z + xyz	y'z	xz	y'z+xz
0	0	0	1	1	0	0	0	0	0	0	0
0	0	1	1	1	0	1	0	1	1	0	1
0	1	0	1	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0	0
1	0	1	0	1	1	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	1	1	0	1	1

xy'z + x'y'z + xyz=(x+x')y'z+ xy'z +xyz=y'z+x(y+y')z=y'z+xz

Question 3

Given the Boolean function

$$F(A, \mathrm{B}, \mathrm{C}, \mathrm{D}) = \sum(0, 1, 6, 7, 10, 11)$$

together with the don't care conditions

$$d(A, B, C, D) = \mathrm{A} \oplus \mathrm{B} \oplus \mathrm{C}. \quad \textbf{(25 pts.)}$$

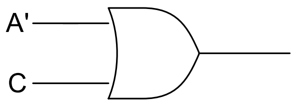
a) Simplify F in sum of products (SOP).
b) Implement F with one NAND gate only.
c) Simplify F in product of sums (POS).
d) Implement F with two NOR gates only.

A	B	C	D	d
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

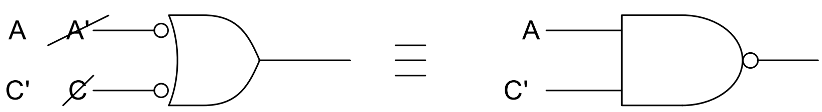
$$d(A, B, C, D) = \sum(2, 3, 4, 5, 8, 9, 14, 15)$$

		CD			
		00	01	11	10
AB	00	1	1	X	X
	01	X	X	1	1
	11	0	0	X	X
	10	X	X	1	1

$$F(A, B, C, D) = A' + C$$

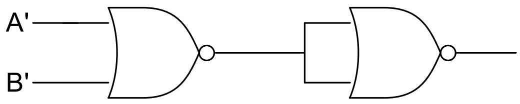


For NAND implementation replace OR gate with invert-OR symbol of NAND.



$$F' = AB$$

$$F = (F')' = A' + B'$$



Question 4

Implement the Boolean function

$$F(A,B,C,D) = C + (A + B')(B' + D)(A' + D)$$

with the minimum number of

a) NAND gates.
b) NOR gates. **(25 pts.)**

$$F(A,B,C,D) = C + (A + B')(B' + D)(A' + D)$$
$$F(A,B,C,D) = C + \left(AB' + AD + B' + B'D\right)(A' + D)$$
$$F(A,B,C,D) = C + \left(AB'D + AD + A'B' + B'D + A'B'D\right)$$

		CD			
		00	01	11	10
AB	00	1	1	1	1
	01	0	0	1	1
	11	0	1	1	1
	10	0	1	1	1

$$F\left(A,B,C,D\right) = A'B' + C + AD \text{ (Sum of Products)}$$
$$F'\left(A,B,C,D\right) = A'BC' + AC'D'$$
$$F\left(A,B,C,D\right) = \left(A + B' + C\right)\left(A' + C + D\right) \text{ (Product of Sums)}$$