# PHYS101 Midterm Exam - Solution Set

Department of Physics

Fall 2015/16 - November 25, 2015

**Fair Use Disclaimer**



**Questions:**

1. Given the vectors $\vec{A} = (4m)\hat{\mathbf{i}} - (3m)\hat{\mathbf{j}}$ and $\vec{B} = (-1m)\hat{\mathbf{i}} + (1m)\hat{\mathbf{j}}$

   (a) Find the vector $\vec{D} = 2\vec{A} - \vec{B}$. (1 P)
   **Solution:**

$$\vec{D} = 2\left[(4m)\hat{\mathbf{i}} - (3m)\hat{\mathbf{j}}\right] - \left[(-1m)\hat{\mathbf{i}} + (1m)\hat{\mathbf{j}}\right] = (9m)\hat{\mathbf{i}} - (7m)\hat{\mathbf{j}}$$

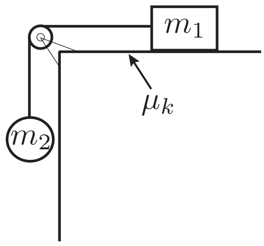
   (b) Find the magnitudes of $\vec{A}$ and $\vec{B}$. (1 P)
   **Solution:**

$$\begin{aligned} |\vec{A}| &= \sqrt{(4m)^2 + (-3m)^2} = \sqrt{25m^2} = 5m \\ |\vec{B}| &= \sqrt{(-1m)^2 + (1m)^2} = \sqrt{2m^2} = \sqrt{2}m \approx 1.41m \end{aligned}$$

   (c) Find the angle between the vector $\vec{D}$ and the positive $x$-axis. (2 P)
   **Solution:**

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-7m}{9m}\right) = -37.9^\circ = 322.1^\circ$$

2. A block of mass $m_1 = 1kg$ on a rough horizontal surface is connected by a cord over a massless, frictionless pulley to a ball of mass $m_2 = 2kg$. The coefficient of kinetic friction between the block and the horizontal surface is $\mu_k = 0.5$.




(a) Draw the free body diagrams for the block and the ball. (3P)

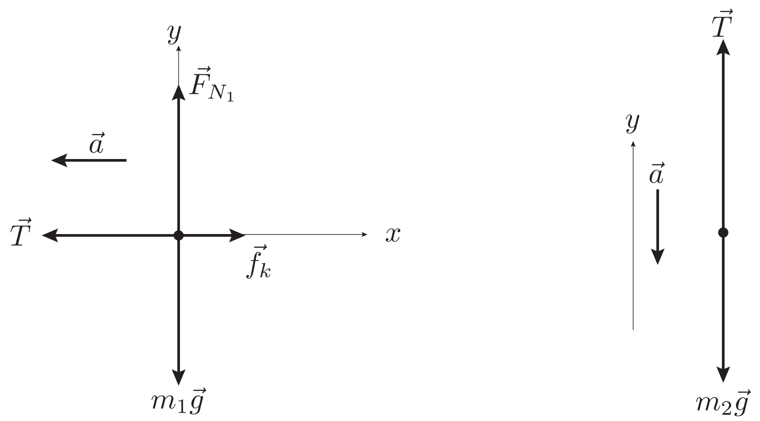
(b) Calculate the magnitude of the acceleration of the block and the ball. (2 P)

(c) Calculate the tension in the cord. (1 P)

**Solution:**

(a) free body diagrams for $m_1$ and $m_2$

free body diagram for $m_1$ free body diagram for $m_2$




(b) We get from the free body diagrams we get:

$$\sum \vec{\mathbf{F}}_1 = (-T + f_k)\hat{\mathbf{i}} + (F_{N_1} - m_1 g)\,\hat{\mathbf{j}} = -m_1 a\hat{\mathbf{i}} \quad (1)$$
$$\sum \vec{\mathbf{F}}_2 = (T - m_2 g)\hat{\mathbf{j}} = -m_2 a\hat{\mathbf{j}} \quad (2)$$

writing (1) in component form gives:

$$-T + f_k = -m_1 a, \quad F_{N_1} - m_1 g = 0 \Longrightarrow F_{N_1} = m_1 g \Longrightarrow f_k = \mu_k F_{N_1} = \mu_k m_1 g$$

so we get

$$-T + \mu_k m_1 g = -m_1 a \quad (3)$$

(2) gives:

$$T - m_2 g = -m_2 a \quad (4)$$

(3) + (4) yields to:

$$\mu_k m_1 g - m_2 g = -m_1 a - m_2 a$$
$$\Longrightarrow a = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g = \frac{2kg - 0.5 \cdot 1kg}{2kg + 1kg} g = \frac{1}{2} g = 4.9 \frac{m}{s^2}$$

(c) So we get for $T$ from equation (4)

$$T = m_2(g - a) = 2kg \left(9.8 \frac{m}{s^2} - 4.9 \frac{m}{s^2}\right) = 9.8N$$

3. The position vector $\vec{\mathbf{r}}(t)$ of a particle moving in the $xy$- plane is given by $\vec{\mathbf{r}}(t) = \left(2t^3 - 5t\right)\hat{\mathbf{i}} + \left(6 - 7t^4\right)\hat{\mathbf{j}}$ with $\vec{\mathbf{r}}(t)$ in meters and $t$ in seconds.

(a) Calculate the position vectors at $t = 0$ and $t = 1s$. (1 P)
**Solution:**

$$\begin{aligned} \vec{\mathbf{r}}(0) &= (6m)\,\hat{\mathbf{j}} \\ \vec{\mathbf{r}}(1s) &= -(3m)\,\hat{\mathbf{i}} - (1m)\,\hat{\mathbf{j}} \end{aligned}$$

(b) Calculate the displacement of the particle between $t = 0$ and $t = 1s$. (1 P)
**Solution:**

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(1s) - \vec{\mathbf{r}}(0) = \left(-(3m)\,\hat{\mathbf{i}} - (1m)\,\hat{\mathbf{j}}\right) - (6m)\,\hat{\mathbf{j}} = -(3m)\,\hat{\mathbf{i}} - (7m)\,\hat{\mathbf{j}}$$

(c) Calculate the average velocity of the particle between $t = 0$ and $t = 1s$. (1 P)
**Solution:**

$$\vec{\mathbf{v}}_{avg} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{-(3m)\,\hat{\mathbf{i}} - (7m)\,\hat{\mathbf{j}}}{1s - 0} = -\left(3\frac{m}{s}\right)\hat{\mathbf{i}} - \left(7\frac{m}{s}\right)\hat{\mathbf{j}}$$

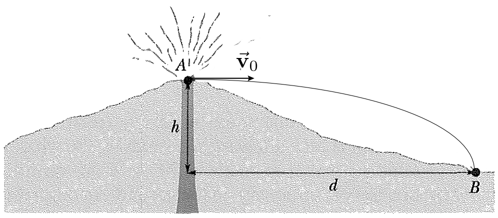
(d) Calculate the instantaneous velocity and the acceleration of the particle at $t = 1s$. (1 P)

$$\begin{aligned} \vec{\mathbf{v}}(t) &= \left(6t^2 - 5\right)\hat{\mathbf{i}} - 28t^3\,\hat{\mathbf{j}} \Longrightarrow \vec{\mathbf{v}}(1s) = \left(1\frac{m}{s}\right)\hat{\mathbf{i}} - \left(28\frac{m}{s}\right)\hat{\mathbf{j}} \\ \vec{\mathbf{a}}(t) &= 12t\,\hat{\mathbf{i}} - 84t^2\,\hat{\mathbf{j}} \Longrightarrow \vec{\mathbf{a}}(1s) = \left(12\frac{m}{s^2}\right)\hat{\mathbf{i}} - \left(84\frac{m}{s^2}\right)\hat{\mathbf{j}} \end{aligned}$$

4. During volcanic eruptions, chunks of solid rock can be blasted out of the volcano; these projectiles are called volcanic bombs. The figure below shows a cross section of Mt. Fuji, in Japan. From the vent A to the foot of the volcano at B, the vertical distance is $h = 3.30km$ and horizontal distance is $d = 940m$. Neglecting air resistance,

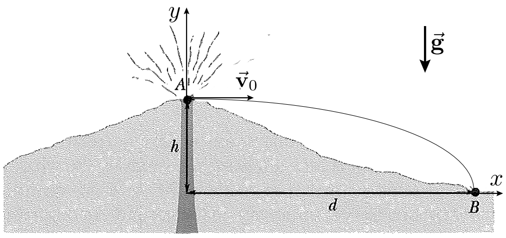
   (a) calculate the time of flight, and (4 P)

   (b) calculate the initial speed of the projectile. (2P)




**Solution:**

First we have to set the coordinate system. If we select the coordinate system as following:




We get for the initial position and the acceleration:

$$\vec{\mathbf{r}}_0 = 3.30km\,\hat{\mathbf{j}} = 3300m\,\hat{\mathbf{i}}$$
$$\vec{\mathbf{a}} = -g\,\hat{\mathbf{j}}$$

(a) In order to calculate the time of flight we first consider the $y$- component of the position vector

$$y(t) = y_0 + v_{y_0}t - \frac{1}{2}gt^2 = y_0 - \frac{1}{2}gt^2$$

The $y$-position at the time of the impact is according to the selection of our coordinate system $y = 0$. So we have to solve the following equation

$$0 = y_0 - \frac{1}{2}gt^2 \Longrightarrow t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2 \cdot 3300m}{9.8\frac{m}{s^2}}} = 25.95s$$

So, the time of flight is: $t = 25.95s$.

(b) As we also know the horizontal distance of the projectile we can easily calculate the initial velocity in this case:

$$x(t) = x_0 + v_0t \Longrightarrow v_0 = \frac{x - x_0}{t} = \frac{940m - 0}{25.95s} = 36.22\frac{m}{s}.$$

So we get for the velocity $\vec{\mathbf{v}}_0 = 36.22\frac{m}{s}\,\hat{\mathbf{i}}$.