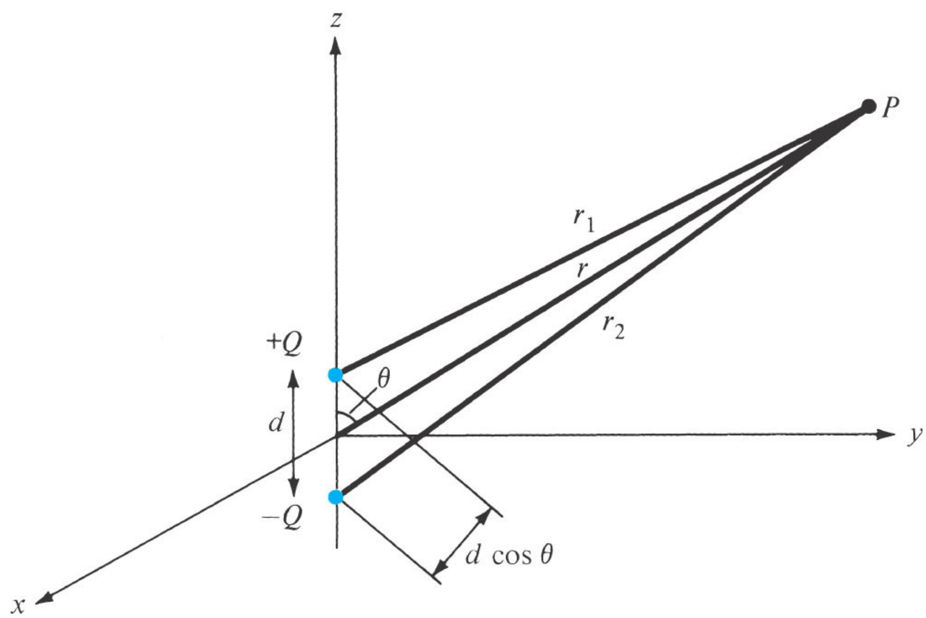
Electric Dipole



An electric dipole is composed of two point charges +Q and –Q separated by a small distance d having the dipole moment vector $\overline{P} = Qd\,\hat{a}_z$.

Determine the potential V and the electric field intensity $\overline{E}$ at an arbitrary point P at a distance r>>d from the dipole (origin).

$$V_p = \frac{Q}{4\pi\varepsilon_0 r_1} + \frac{-Q}{4\pi\varepsilon_0 r_2}$$

If r>>d, r_1, r_2 and r become parallel to each other and

$$r_1 \approx r - \frac{d}{2}\cos\theta$$

$$r_2 \approx r + \frac{d}{2}\cos\theta$$

So,

$$V_p = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r-\frac{d}{2}\cos\theta} - \frac{1}{r+\frac{d}{2}\cos\theta}\right] = \frac{Q}{4\pi\varepsilon_0 r}\left[\frac{1}{1-\frac{d\cos\theta}{2r}} - \frac{1}{1+\frac{d\cos\theta}{2r}}\right]$$

Since,

$(1+\varepsilon)^n \approx 1+n\varepsilon$ for $|\varepsilon| << 1$,

$$\frac{1}{1-\frac{d\cos\theta}{2r}} \approx 1+\frac{d\cos\theta}{2r}$$

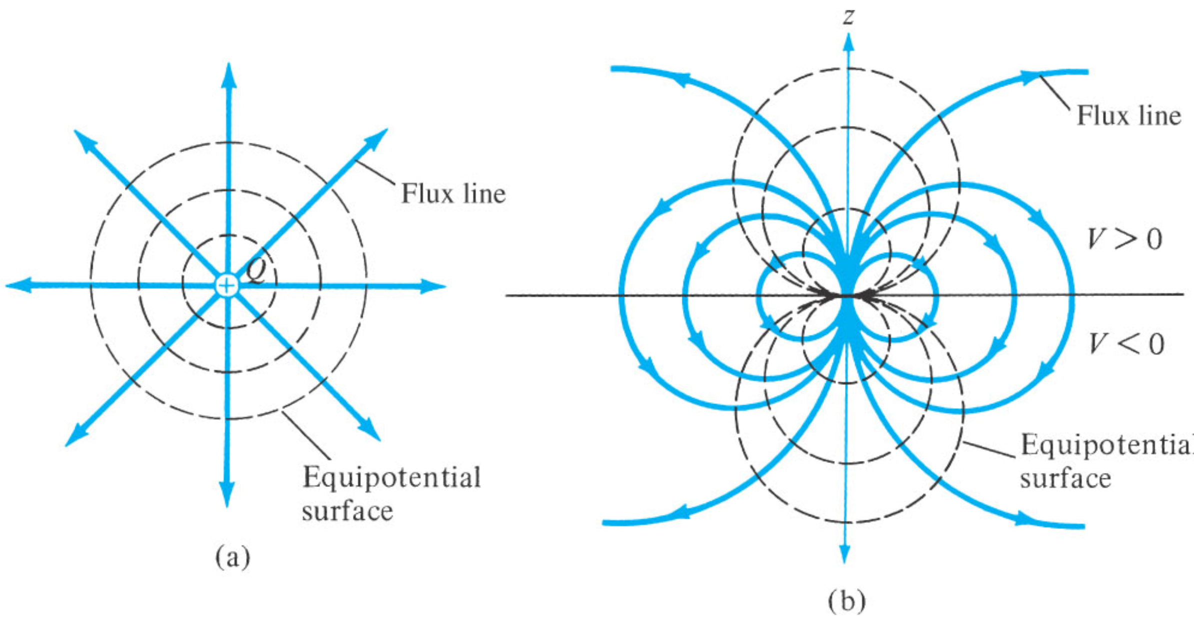
$$\frac{1}{1+\frac{d\cos\theta}{2r}} \approx 1-\frac{d\cos\theta}{2r}$$

$$V_P \approx \frac{Q}{4\pi\varepsilon_0 r}\left[1+\frac{d\cos\theta}{2r} - \left(1+\frac{d\cos\theta}{2r}\right)\right] = \frac{Q}{4\pi\varepsilon_0 r}\frac{d\cos\theta}{r} = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}$$

The electric field:

$$\overline{E}_P = -\nabla V_P = -\left(\frac{\partial V_P}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V_P}{\partial\theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V_P}{\partial\phi}\hat{a}_\phi\right)$$

$$\overline{E}_P = \frac{P}{4\pi\varepsilon_0}\left(\frac{2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta}{r^3}\right)$$



Equipotential surfaces for **(a)** a point charge and **(b)** an electric dipole.