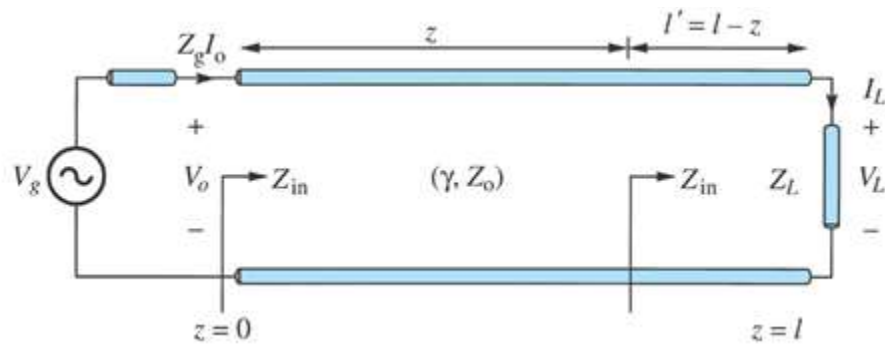


## FINITE LENGTH LINE TERMINATED IN A GENERAL $Z_L$



Reflections occur when  $Z_L \neq Z_0$ .

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V^+ + V^-$$

$$I(z) = I_0^+ e^{-\gamma z} - I_0^- e^{+\gamma z} = I^+ - I^-$$

Reflection Coefficient:

$$\Gamma = \frac{\text{Reflected Voltage (or Current) at pt. } z}{\text{Forward Voltage (or Current) at pt. } z} = \frac{V^-}{V^+} = \frac{I^-}{I^+}$$

The load reflection Coefficient

$$\Gamma_L = \frac{V^-(z=l)}{V^+(z=l)} = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

Defining  $z = l - z'$ , we express  $V$  and  $I$  as:

$$V = V_0^+ e^{-\gamma(l-z')} + V_0^- e^{\gamma(l-z')}$$

$$I = \frac{V_0^+}{Z_0} e^{-\gamma(l-z')} - \frac{V_0^-}{Z_0} e^{\gamma(l-z')} \quad \text{or,}$$

$$V = V_0^+ e^{-\gamma l} \left[ e^{\gamma z'} + \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-\gamma z'} \right]$$

$$I = \frac{V_0^+}{I_0} e^{-\gamma l} \left[ e^{\gamma z'} - \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-\gamma z'} \right]$$

So,

$$V = V_0^+ e^{-\gamma l} \left[ e^{\gamma z'} + \Gamma_L e^{-\gamma z'} \right]$$

$$I = \frac{V_0^+}{I_0} e^{-\gamma l} \left[ e^{\gamma z'} - \Gamma_L e^{-\gamma z'} \right]$$

Since,

$$V^+ = V_0^+ e^{-\gamma z} = V_0^+ e^{-\gamma l} e^{\gamma z'}$$

$$V^- = V_0^- e^{\gamma z} = V_0^- e^{\gamma l} e^{-\gamma z'}$$

$$\Gamma = \frac{V^-}{V^+} = \frac{V_0^-}{V_0^+} e^{2\gamma l} e^{-2\gamma z'} = \Gamma_L e^{-2\gamma z'}$$

So the reflection coefficient at any point on the transmission line:

$$\Gamma = \Gamma_L e^{-2\gamma z'} = \Gamma_L e^{-2\alpha z'} e^{-j2\beta z'}$$

Or,

$$\Gamma = \Gamma_L e^{-2\gamma z'} = \Gamma_L e^{-2\alpha z'} \angle -2\beta z'$$

If,

$$\Gamma = |\Gamma| \angle \phi \quad \Gamma_L = |\Gamma_L| \angle \phi_L$$

Then

$$\Gamma = |\Gamma| \angle \phi = |\Gamma_L| e^{-2\alpha z'} \angle \phi_L - 2\beta z'$$

$$|\Gamma| = |\Gamma_L| e^{-2\alpha z'}$$

$$\phi = \phi_L - 2\beta z'$$

As a special point, take  $z' = l$

$$\Gamma_{in} = \Gamma_L e^{-2\gamma l} \angle \phi_L - 2\beta l$$

$\Gamma_L$  CAN BE CALCULATED FROM  $Z_L$  AND  $Z_0$

If we put  $z' = 0$ ,  $V = V_L$  and  $I = I_L$  so,

$$V_L = V_0^+ e^{-\gamma l} (1 + \Gamma_L)$$

$$I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} (1 - \Gamma_L)$$

### STANDING WAVE PATTERN

Consider a lossless line, then  $\gamma = j\beta$ ,  $\alpha = 0$ .

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} = V_0^+ e^{-j\beta z} \left( 1 + \frac{V_0^-}{V_0^+} e^{j2\beta z} \right)$$

But,

$$\Gamma_L = \frac{V_0^-}{V_0^+} e^{j2\beta l}$$

So,

$$\frac{V_0^-}{V_0^+} = \Gamma_L e^{-j2\beta l} \quad \text{and}$$

$$V(z) = V_0^+ e^{-j\beta z} \left( 1 + \Gamma_L e^{j2\beta(z-l)} \right)$$

$$\text{With } \Gamma_L = |\Gamma| e^{j\phi_L}$$

$$V(z) = V_0^+ e^{-j\beta z} \left( 1 + |\Gamma_L| e^{j(2\beta z - 2\beta l + \phi_L)} \right)$$

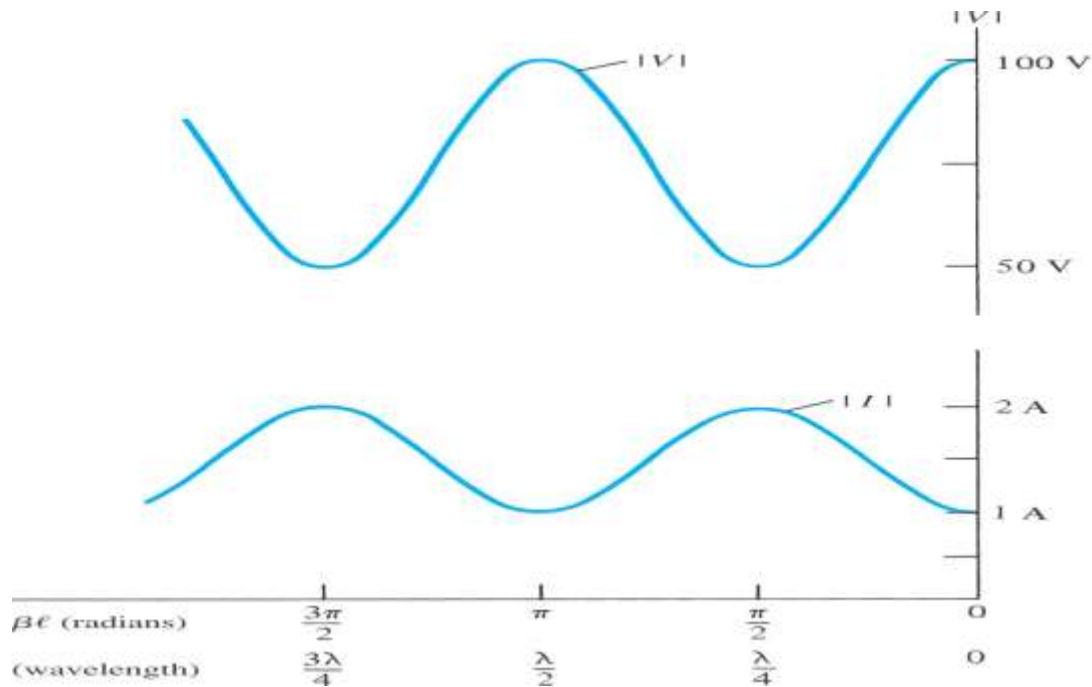
**The Standing-Wave Pattern (Voltage):**

$$|V(z)| = |V_0^+| \left| \left( 1 + |\Gamma_L| e^{j(2\beta z - 2\beta l + \phi_L)} \right) \right|$$

$$|V(z)| = |V_0^+| \left| \left( 1 + |\Gamma_L| e^{-j(2\beta(l-z)) - \phi_L} \right) \right|$$

$$|V(z)| = |V_0^+| \left| \left( 1 + |\Gamma_L| e^{-j(2\beta z' - \phi_L)} \right) \right|$$

$$|V(z)| = |V_0^+| \left( 1 + |\Gamma_L|^2 + 2|\Gamma_L| \cos(2\beta z' - \phi_L) \right)^{1/2}$$



Maxima of Voltage:

$$2\beta z'_{\max, m} - \phi_L = 2\pi m, \quad m = 0, 1, \dots$$

Minima of Voltage:

$$2\beta z'_{\min, n} - \phi_L = (2n - 1)\pi, \quad n = 1, 2, \dots$$

Current Standing Wave Pattern:

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z} = \frac{V_0^+}{Z_0} e^{-j\beta z} \left( 1 - \frac{V_0^-}{V_0^+} e^{2j\beta z} \right)$$

$$|I(z)| = \frac{|V_0^+|}{|Z_0|} \left( 1 + |\Gamma_L|^2 - 2|\Gamma_L| \cos(2\beta z' - \phi_L) \right)^{1/2}$$

Current maxima occur at voltage minima and current minima occur at voltage maxima.

### Standing -Wave Ratio (SWR)

$$|V(z)|_{\max} = V_0^+ (1 + |\Gamma_L|) \quad |V(z)|_{\min} = V_0^+ (1 - |\Gamma_L|)$$

$$SWR = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$SWR \geq 1$$

If  $\Gamma = 0$ ,  $SWR = 1$  (no reflections exists)

If  $\Gamma = 1$ ,  $SWR = \infty$

$$|\Gamma_L| = \frac{SWR - 1}{SWR + 1}$$

i)  $Z_L = 0$  (short circuit)  $\rightarrow \Gamma_L = -1 \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$

ii)  $Z_L = \infty$  (open circuit)  $\rightarrow \Gamma_L = +1 \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$



**iii)**  $Z_L = Z_0$  (*matched circuit*)  $\rightarrow \Gamma_L = 0 \rightarrow SWR = 1$

**iv)**  $Z_L = jX_L$  (*reactive load*)  $\rightarrow \Gamma_L = e^{j\theta_L} \rightarrow |\Gamma_L| = 1 \rightarrow SWR = \infty$