

## Electrostatic Potential Energy

When a source is connected to a capacitor, it spends energy to charge up the capacitor.

This charging energy is stored in the dielectric medium in the form of electrostatic potential energy.

The amount of stored energy  $W_e$ , is related to  $Q$ ,  $C$  and  $V$ .

Under the influence of the electric field in a dielectric medium between the two conductors, charge  $q$  accumulated on one of the conductors and  $-q$  on the other conductor. The voltage  $V$  across the capacitor:

$$V = \frac{q}{C}$$

The amount of work done  $dW_e$  required to transfer an additional amount of charge  $dQ$  is:

$$dW_e = Vdq = \frac{q}{C}dq$$

The amount of work required to charge an uncharged capacitor from 0 to charge  $Q$ :

$$W_e = \int_0^Q Vdq = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} \quad (J)$$

(If an uncharged capacitor is charged up from 0 to  $Q$ ,  $W_e$  will be the amount of work required).

Substitute  $C = \frac{Q}{V}$  in  $W_e = \frac{1}{2} \frac{Q^2}{C}$  gives

$$W_e = \frac{1}{2} CV^2 \quad (J),$$

where  $V$  is the final voltage.

For the parallel plate capacitor,

$$C = \frac{\epsilon A}{d}, \quad W_e = \frac{1}{2} \frac{\epsilon A}{d} V^2 \quad \text{and} \quad V = Ed \quad \text{results}$$

$$W_e = \frac{1}{2} \frac{\epsilon A}{d} E^2 d^2$$

$$W_e = \frac{1}{2} \epsilon E^2 A d$$

(  $dA$  ) is the volume of the capacitor.

The electrostatic energy density:

$$dw_e = \frac{W_e}{Ad} = \frac{1}{2} \epsilon |E|^2 \text{ (J / m}^3\text{)}$$

Although this expression is derived for the parallel plate capacitor, it is valid for any other dielectric medium in an electric field  $\bar{E}$ .

Furthermore, the total electrostatic energy in volume  $\mathcal{V}$  is:

$$W_e = \frac{1}{2} \int_{\mathcal{V}} \epsilon |E|^2 dv \text{ (J) or,}$$

$$W_e = \frac{1}{2} \int_{\mathcal{V}} \rho V dv \text{ (J)}$$

for the continuous charge distribution  $\rho$ . Here the assumption is that the charges are at  $\infty$ .

Also note that the work done is equal to the electrostatic energy stored in a region of space.