SOLUTION HW 01

GIVEN DATA:-

Width of Microstrip Line, $W = 3.058mm$

Relative Permittivity of FR-4, $\varepsilon_r = 4.4$

Thickness of Microstrip Line, $t = 0.1mm$

Height of FR-4 Substrate, $h = 1.7mm$

Load Impedance, $Z_L = 400\Omega$

Power at Input terminal, $P_{in} = 200W$

Frequency, $f = 2.45\,GHz$

REQUIRED:-

(a) Find the characteristic Impedance Z_o of the $W = 3.058$ mm Line.

As W/h ratio results in value greater than 1, So, the characteristic Impedance is given by,

$$Z_o = \frac{120\pi}{\sqrt{\varepsilon_{reff}}\left[W/h + 1.393 + 0.667\ln\left(W/h + 1.444\right)\right]}$$

$W = 3.058mm$, $h = 1.7mm$, $\varepsilon_{reff} = ?$

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[\frac{1}{\sqrt{1 + 12\,h/W}}\right]$$

$$\varepsilon_{reff} = \frac{4.4 + 1}{2} + \frac{4.4 - 1}{2}\left[\frac{1}{\sqrt{1 + 12\left(1.7/3.058\right)}}\right]$$

$$\varepsilon_{reff} = 2.7 + 1.7\,(0.36105)$$

$$\boxed{\varepsilon_{reff} = 3.3138}$$

Now

$$Z_0 = \frac{120\pi}{\sqrt{3.3138}\left[\frac{3.058}{1.7} + 1.393 + 0.667\ln\left(\frac{3.058}{1.7} + 1.444\right)\right.}$$

$$Z_0 = \frac{120\pi}{1.8203\left[1.7988 + 1.393 + 0.667\ln\left(1.7988 + 1.444\right)\right]}$$

$$Z_0 = \frac{120\pi}{7.2383}$$

$$\boxed{Z_0 = 52.05\,\Omega}$$

The required Characteristic Impedance.

(b) Calculate the load reflection Coefficient.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\therefore Z_L = 400\,\Omega$
$Z_0 = 52.05\,\Omega$

$$\Gamma_L = \frac{400 - 52.05}{400 + 52.05}$$

$$\boxed{\Gamma_L = 0.7697}$$ The Load reflection coefficient.

(c) How much of the power is delivered to the load?

Power delivered to load highly depends upon the reflection coefficient;

P_L = Power delivered to the load

$$P_L = P_{in}(1 - |\Gamma_L|^2)$$

$|\Gamma_L|^2 = 0.59$

$$P_L = 200(1 - |0.7697|^2)$$

59 % of the power is reflected back.

$$P_L = 200(0.4075)$$

41% of the power is delivered to the load.

$$\boxed{P_L = 81.51\ W}$$

The power delivered to the load.

(d) Calculate the characteristic impedance Z_o' of the quarter wavelength long line that could be used to match the load and increase the power delivered to the load.

For $\lambda/4$ section,

$$Z_{in} = \frac{(Z_o^1)^2}{Z_L}$$

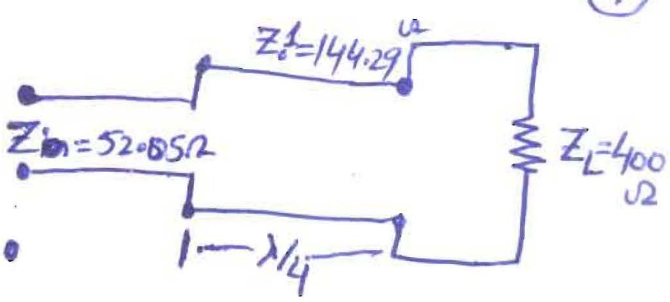
or,

$$(Z_o^1)^2 = Z_{in} Z_L$$

$$Z_o^1 = \sqrt{Z_{in} \cdot Z_L}$$

$$Z_0^1 = \sqrt{400 \times 52.05}$$

$$\boxed{Z_0^1 = 144.29\,\Omega}$$



↳ $Z_0^1 = 144.29\,\Omega$ should be the impedance in order to match $52.05\,\Omega$ line to $400\,\Omega$ load.

ⓔ Find width of $\lambda/4$ section.

As,

$$\frac{W}{h} = \frac{8e^A}{e^{2A}-2} \qquad \text{for } \frac{W}{h} \leq 2$$

Where

$$A = \pi\sqrt{2(\epsilon_r+1)}\,\frac{Z}{\eta_0} + \frac{\epsilon_r-1}{\epsilon_r+1}\left(0.23+\frac{0.11}{\epsilon_r}\right)$$

$$A = \pi\sqrt{2(4.4+1)} \cdot \frac{144.29}{120\pi} + \frac{4.4-1}{4.4+1}\left(0.23+\frac{0.11}{4.4}\right)$$

$$A = 10.319(0.3827) + (0.6296 \times 0.255)$$

$$A = 3.949081 + 0.160548$$

$$\boxed{A = 4.1096}$$

Now

$$W = \frac{8e^A}{e^{2A}-2} \times h$$

$$W = \frac{8e^{4.1096} \times 1.7}{e^{2(4.1096)} - 2}$$

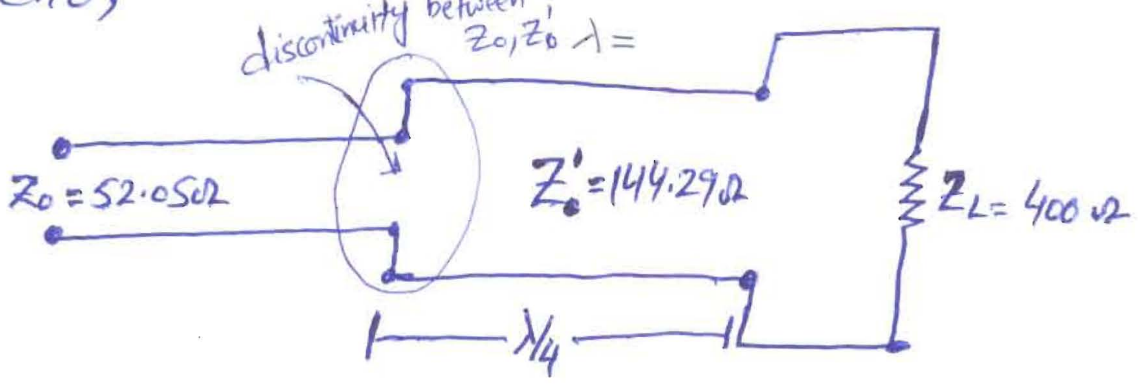
$$W = \frac{828.5438}{3711.5319}$$

$$\boxed{W = 0.2232 \text{ mm}}$$

The required width for $\lambda/4$ transmission line.

(f) Place a section $l = \lambda/4$ of impedance $Z_0' = 144.29\Omega$ between the load and the line with impedance $Z_0 = 52.05\Omega$. Calculate the input impedance by using the general expression of Z_{in} and the new reflection coefficient at the discontinuity of Z_0 and Z_0'.

For simplification, let's draw the overall scenerio,



As

$$Z_{in} = \frac{(Z_0')^2}{Z_L}$$

We know, $Z_0' = 144.29\Omega$ and $Z_L = 400\Omega$

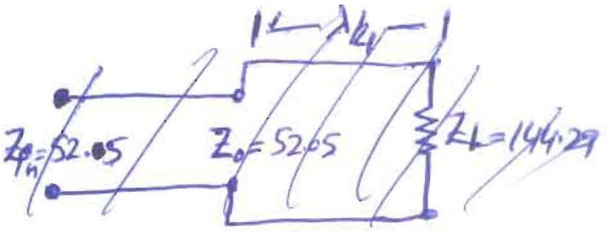
$$Z_{in} = \frac{(144.29)^2}{400}\ \Omega$$

$$\boxed{Z_{in} = 52.05\ \Omega} \longrightarrow 1$$

Now

Reflection coefficient at the discontinuity between Z_o and Z_o', should be;

$$\Gamma(z=\lambda/4) = \frac{Z_{L} - Z_o}{Z_L + Z_o}$$



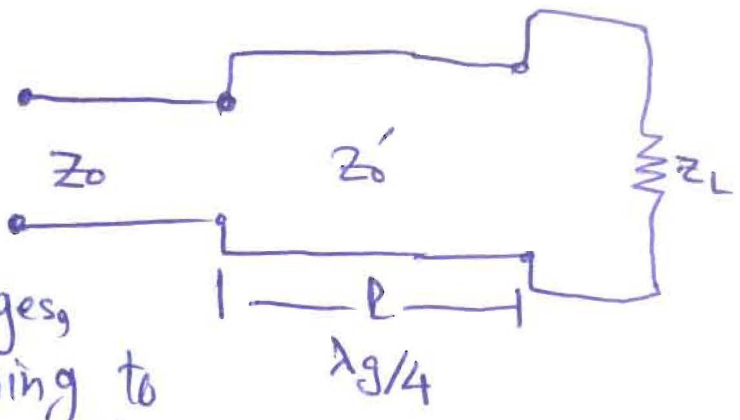
↳ Where at this point $Z_L = Z_{in}$ as we found in part (f). (Equation 1)

$$\Gamma(z=\lambda/4) = \frac{Z_{in} - Z_o}{Z_{in} - Z_o} \Rightarrow \frac{52.05 - 52.05}{52.05 - 52.05}$$

$$\boxed{\Gamma(z=\lambda/4) = 0}$$

(h) What is the power delivered to the load when the frequency is changed to 3GHz.

All the previous calculations were for frequency $f = 2.45GHz$.



When the frequency changes, the first thing that is going to effect is the length "l" as it is factor of wavelength.

So,

For $f = 2.45GHz$

$$\lambda_o = c_o/f = \frac{3\times10^8}{2.45\times10^9}$$

$\lambda_o = 0.1224m$ or $\boxed{\lambda_o = 122.4mm}$

Now

$$\lambda g = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} = \frac{122.4}{\sqrt{3.3138}}$$

$$\boxed{\lambda g = 67.24\,mm}$$

and

Hence

$$\boxed{\lambda g/_4 = 16.80\,mm}$$

Incase the frequency changed to 3GHz.

then,

$$\lambda_{o_{new}} = \frac{c_o}{f} = \frac{3\times10^8}{3\times10^9} \Rightarrow 0.1\,m$$

$$\boxed{\lambda_{o_{new}} = 100\,mm}$$

Similarly;

$$\lambda_{g_{new}} = \frac{\lambda_{o_{new}}}{\sqrt{\varepsilon_{reff}}} \Rightarrow \frac{100}{\sqrt{3.3138}}$$

$$\boxed{\lambda g_{new} = 54.93\,mm}$$

Butt the change in frequency caused a change in length of the matching transformer, to find that we need to do;

$$\frac{\lambda g_{new}}{\lambda g/4} = 3.266$$

So, Now instead of $\lambda g/4$, the length will be $\lambda g/3.266$.

Hence it is not $\lambda g/4$, So we cannot use the equations of Quarter wavelength,
So,

~~$Z_{in} = Z_o \cdot Z_L + g$~~

$$Z_{in} = Z_o \cdot \left[\frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right] \rightarrow (1)$$

Now,

$$\beta l = \frac{2\pi}{\cancel{\lambda g}} \cdot \frac{\cancel{\lambda g}}{3.266}$$

$$\boxed{\beta l = 1.9238}$$

and

$$\tan \beta l = \tan(1.9238)$$

$$\boxed{\tan \beta l = -2.7141 \text{ rad}}$$

Putting back values in Equ (1), we get

$$Z_{in} = 73.795 \angle 0.6636 \text{ rad}$$

Now

Finding reflection Coefficient;

$$\Gamma(z' = \lambda/4) = 0.3846 \angle 1.074 \text{ rad}$$

Finding the power delivered to the load,

$$P_L = P_{in}\left(1 - |\Gamma(z' = \lambda/4)|^2\right)\left(1 - |\Gamma_L|^2\right)$$

$$\boxed{P_L = 132.83\,W}$$

the required Answer

x