HOME WORK 2 SOLUTION

Q1:- GIVEN:-

- Air Filled Rectangular Waveguide
- Cut-off frequency for TE_{10}, $f_{c_{1,0}} = 5GHz$
- Cutoff frequency for TE_{01}, $f_{c_{01}} = 12GHz$

@ The dimension of the waveguide.

As the waveguide is air filled, the value of $\varepsilon_r = 1$.

The cut off frequency formula is given by;

$$f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

where in case of TE_{10},

$m=1$, $n=0$, $f_{c_{10}} = 5GHz$

$$5\times10^9 = \frac{3\times10^8}{2\pi(1)}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{0\pi}{b}\right)^2}$$

$$5\times10^9 = 0.478\times10^8\sqrt{\left(\frac{\pi}{a}\right)^2}$$

$$5\times10^9 = 0.478\times10^8\left(\frac{\pi}{a}\right)$$

$$a = \frac{0.478\times10^8\,\pi}{5\times10^9}$$

$$\boxed{a = 0.03m}$$

In case of TE_{01},

where $m=0$, $n=1$ and $f_c = 12GHz$.

$$f_c = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$12GHz = 0.478\times10^8\sqrt{\left(\frac{\pi}{b}\right)^2}$$

$$12\times10^9 = 0.478\times10^8\left(\frac{\pi}{b}\right)$$

$$b = \frac{0.478\times10^8\,\pi}{12\times10^9}$$

$$\boxed{b = 0.0125\,m}$$

(b) The cut off frequency of the next three higher TE modes.

$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)}$$

for TE_{11}

$$f_c = 0.478\times10^8\sqrt{\left(\frac{\pi}{0.03}\right)^2+\left(\frac{\pi}{0.0125}\right)^2}$$

$$f_c = 0.478\times10^8\sqrt{74056.55}$$

$$f_c = 130.07\times10^8$$

$$\boxed{f_c = 13\,GHz}$$

Similarly, the cutoff frequencies are; (3)

$TE_{20} = 10\,GHz$

$TE_{02} = 24\,GHz$

$TE_{21} = 15.62\,GHz$

$TE_{12} = 24.5\,GHz$

$TE_{30} = 15\,GHz$

$TE_{03} = 36\,GHz$

$TE_{31} = 19.2\,GHz$

$TE_{13} = 36.345\,GHz$

$TE_{40} = 20\,GHz$

So, the next three higher TE_{mn} modes are,

TE_{11}, TE_{30} and TE_{21}.

(c) When $\varepsilon_r = 2.25$, $\mu_r = 1$

$f_c = ?$ for TE_{11}

$$f_c = \frac{c}{2\pi\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{3\times 10^8}{2\pi\sqrt{2.25}}\sqrt{\left(\frac{\pi}{0.03}\right)^2 + \left(\frac{\pi}{0.0125}\right)^2}$$

$$f_c = 0.318\times10^8 \sqrt{\left(\frac{\pi}{0.03}\right)^2 + \left(\frac{\pi}{0.0125}\right)^2}$$ (4)

$$f_c = 0.318\times10^8 \sqrt{74056.55}$$

$$f_c = 86.5\times10^8$$

$$f_c = 8.65\times10^9$$

$$\boxed{f_c = 8.6\,GHz}$$

X

Q2:- GIVEN:-

Rectangular Waveguide

Broader Wall, $a = 2.5cm = 0.025m$

Narrow Wall, $b = 1\,cm = 0.01m$

$\mu = \mu_0$ and $\varepsilon = 4\varepsilon_0$

Frequency of operation < 15.2 GHz

(a) How many TE and TM modes can be propagated, Calculate the cut off frequencies of the modes.

$$f_c = \frac{c}{2\pi\sqrt{4\varepsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{c}{4\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Now finding TE_{mn}, TM_{mn} modes that propagates; and their cut off frequencies;

$$TE_{10} = TM_{10} = \frac{3\times10^8}{4\pi}\sqrt{\left(\frac{\pi}{0.025}\right)^2 + 0}$$

$$TE_{10} = TM_{10} = 0.2388\times10^8\sqrt{(125.6)^2}$$

$$TE_{10} = 30\times10^8$$

$$TE_{10} = 3GHz < 15.2GHz$$

So, this mode will propagate.

Similarly;

$$TE_{01} = TM_{01}$$

$$f_c = \frac{c}{4\pi}\sqrt{0 + \left(\frac{\pi}{b}\right)^2}$$

$$f_c = 0.2388\times10^8\sqrt{\left(\frac{\pi}{0.01}\right)^2}$$

$$f_c = 7.5\,GHz$$

This mode will also popagate.

Following is the table of all those modes and their cutoff frequencies, which will propagate and not propagate.

$TE_{mn} = TM_{mn}$ for $m = n \leq 5$ Frequency in GHz

	n=0	n=1	n=2	n=3	n=4	n=5
m=0		7.5	15	>15.2	>15.2	>15.2
m=1	3	8.07	22.68	>15.2	>15.2	>15.2
m=2	6	9.6	16.15	>15.2	>15.2	>15.2
m=3	9	11.7	17.5	>15.2	>15.2	>15.2
m=4	12	14.15	19.2	>15.2	>15.2	>15.2
m=5	15	16.75	21.2	>15.2	>15.2	>15.2

All the shaded modes, whose cutoff frequency is less than 15.2 GHz are the modes which will propagate only.

These are

$TE_{mn} = TM_{mn}$

$\Rightarrow$ 10, 20, 30, 40, 50, 01, 11, 21, 31, 41, 02.

and their alternative case as well

$\Rightarrow$ 01, 02, 03, 04, 05, 12, 13, 14,

(b) Calculate the Phase Constant, phase Velocity and the wave impedance for the dominant mode TE_{10} operating at 15 GHz.

Phase Constant, $\beta = ?$

$$\beta = \sqrt{K^2 - Kc^2}$$

Now

$$K = \omega\sqrt{\mu\varepsilon}$$

or

$$K = \frac{\omega\sqrt{\varepsilon_r \mu_r}}{c}$$

$$k = \frac{2\pi f\sqrt{4}}{3\times10^8} \Rightarrow \frac{2\pi\times15\times10^9\times2}{3\times10^8}$$

$$\boxed{K = 200\pi}$$

Where

$$Kc = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$Kc = \sqrt{\left(\frac{\pi}{0.025}\right)^2 + 0}$$

$$\boxed{Kc = 40\pi}$$

So,

$$\beta = \sqrt{K^2 - Kc^2} \Rightarrow \sqrt{(200\pi)^2 - (40\pi)^2}$$

$$\boxed{\beta = 615.3 \text{ rad/m}}$$

Now

Phase Velocity, $V_p = ?$

$$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 15 \times 10^9}{615.3}$$

$$\boxed{V_p = 153 \times 10^6 \, m/s}$$

Now;

Wave Impedance,

$$Z = \frac{K\eta}{\beta}$$

$$Z = \frac{(200\pi)\left(\frac{120\pi}{\sqrt{4}}\right)}{615.3}$$

$$\boxed{Z = 192.38 \, \Omega}$$