Department of Physics
PHYS102 - Physics II
Midterm Examination
Spring 2015-16
(April 11, 2016)

Solution

Name & Surname:	Number:	Group:	Signature:

1. The exam includes 4 questions.
2. You are not allowed to use any source of information.
3. All electronic devices are forbidden to be used during the exam.
4. Your mobile phone must be turned off before the exam starts.
5. You may use an ordinary calculator.
6. Duration of the exam is 90-minutes.

Good Luck

P1: (10 pts)	P2: (10 pts)	P3: (10 pts)	P4: (10 pts)	Total: (40 pts)

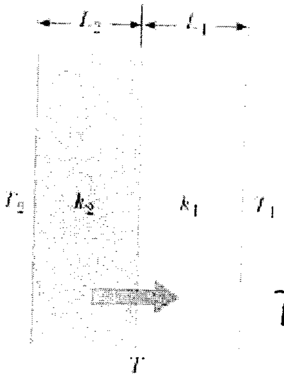
Some useful formulas:

$W = \int P dV$	$\Delta S = \int \frac{dQ}{T}$	First Law: $\Delta U = \Delta E_{int} = Q - W$
$\Delta U = \Delta E_{int} = nC_v \Delta T$	$\vec{E} = k \int \frac{dq}{r^2} \hat{r},$	$1m^3 = 10^3 L$
$\lambda = \frac{q}{L}$	$1atm = 1.01 \times 10^5 pa$	$\vec{F} = q\vec{E}$
$\vec{F} = k_e \frac{q_1 q_2}{r^2} \hat{r}$	$PV = nRT$ $1atm.L = 101\,J$	*Adiabatic*: $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$
$R = 8.314 \frac{J}{mol.K}$	$H = P_{cond} = \frac{A(T_H - T_C)}{\sum \frac{L_i}{k_i}}$	$H = P_{cond} = \frac{Q}{t}$
diatomic: $\gamma = \frac{7}{5}$ $C_p = \frac{7}{2} R, \quad C_v = \frac{5}{2} R$	*monatomic*: $\gamma = \frac{5}{3}$ $C_p = \frac{5}{2} R, \quad C_v = \frac{3}{2} R$	$e = \varepsilon = \frac{W}{Q_H} = 1 - \frac{\lvert Q_L \rvert}{Q_H}$

P1: Thermodynamics: (4+2+4 pts)

Heat is transferred at a steady rate through a composite slab made up of two different materials with different thicknesses and different thermal conductivities but the same areas as shown in the Figure. (Note that $T_1 = 5°\text{C}, T_2 = 30°\text{C}, L_1 = 4mm, L_2 = 6mm, k_1 = 16\frac{W}{m.K}$, and $k_2 = 24\frac{W}{m.K}$ with the area of the slabs given as $A_1 = A_2 = A = 50\ cm^2$.)

a) What is the rate of heat transferred by the slabs?
b) How much energy is transferred by the slabs in 2 hours?
c) What is the steady-state temperature T at the interface (between) of the two slabs?



a) $$P_{cond} = \frac{A(T_H - T_c)}{\frac{l_1}{k_1} + \frac{l_2}{k_2}} = \frac{50\times10^{-4} m^2 (25) K}{\frac{4\times10^{-3} m}{16\ W/m.K} + \frac{6\times10^{-3} m}{24\ W/m.K}}$$

$$P_{Cond} = 250\ W$$

b) $$P_{Cond} = \frac{Q}{t} \Rightarrow Q = P_{Cond} \cdot t = 250 \frac{J}{s} \times 2 \times 3600 s$$

$$Q = 18\times10^{5} J$$

c) $$P^{(1)}_{Cond} = P_{Cond} = P^{(2)}_{Cond}$$

$$\Rightarrow \frac{(T_2 - T)\, 50\times10^{-4}}{\frac{6\times10^{-3}}{24}} = 250 \Rightarrow T_2 - T = 12.5$$

$$\Rightarrow T = T_2 - 12.5 = 17.5\ °C$$

second method:

a) $R^{(1)}_t = \frac{l_1}{Ak_1} = 0.05 \frac{K}{W}$, $R^{(2)}_t = \frac{l_2}{Ak_2} = 5\times10^{-2} = 0.05 \frac{K}{W}$

$$R^{(total)}_t = R^{(1)}_t + R^{(2)}_t = 0.05 + 0.05 = 0.10 \frac{K}{W} \Rightarrow P_{Cond} = \frac{T_H - T_c}{R^{(total)}_t} = \frac{25}{0.1} = 250W$$

b) $$\frac{T_2 - T}{R^{(2)}_t} = \frac{T - T_1}{R^{(1)}_t} \Rightarrow 0.05(T_2 - T) = 0.05(T - T_1) \Rightarrow T = 17.5\ °C$$

c) the same

P2: Thermodynamics: (4+2+2+2 pts)

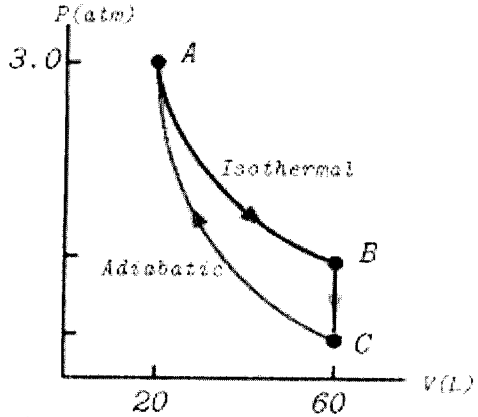
2.0 moles of a diatomic ideal gas is taken through the cycle shown in the figure. Use the information given on the figure to answer the following questions:

a) What are the pressures of the gas at points B and C?

b) What is the work done by the gas in the process AB?

c) What is the work done by the gas in the process CA?

d) What is the change in the entropy of the gas in the process BC?



a) $P_A V_A = P_B V_B \Rightarrow P_B = P_A \frac{V_A}{V_B} = 3\,atm\,\frac{20L}{60L} = 1\,atm$

and $P_A V_A^{\gamma} = P_C V_C^{\gamma} \Rightarrow P_C = P_A\left(\frac{V_A}{V_C}\right)^{\gamma} = 3atm\left(\frac{20L}{60L}\right)^{7/5} = 0.64\,atm$

b) $W_{AB} = nRT_{AB}\ln\left(\frac{V_B}{V_A}\right) = P_A V_A \ln\left(\frac{V_B}{V_A}\right) = 60atm.L\ \ln\left(\frac{60L}{20L}\right)$

$W_{AB} = 60\times 101\times \ln(3)\,J = 6657.6\,J$

c) $W_{CA} = -nC_v(T_A - T_C) = -n\frac{5}{2}R(T_A - T_C) = -\frac{5}{2}(nRT_A - nRT_C)$

$W_{CA} = -\frac{5}{2}(P_A V_A - P_C V_C) = -\frac{5}{2}(60 - 0.64\times 60)atm.L$

$W_{CA} = -\frac{5}{2}(60 - 0.64\times 60)\ 101\,J = -5454\,J$

d) $\Delta S_{BC} = nC_v\ln\left(\frac{T_C}{T_B}\right) + \underbrace{nR\ln\Big(\underset{1}{\cancel{\frac{V_C}{V_B}}}\Big)}_{0} = nC_v\ln\left(\frac{T_C}{T_B}\right)$

$\Delta S_{BC} = n\frac{5}{2}R\ln\left(\frac{P_C V_C/nR}{P_B V_B/nR}\right) = \frac{5}{2}nR\ln\left(\frac{P_C}{P_B}\right) = \frac{5}{2}nR\ln\left(\frac{0.64}{1}\right)$

$\Delta S_{BC} = \frac{5}{2}\times 2\times 8.314\ln(0.64)\ \frac{J}{K} = -18.55\,\frac{J}{K}$

P3: Thermodynamics: (2+2+2+4 pts)

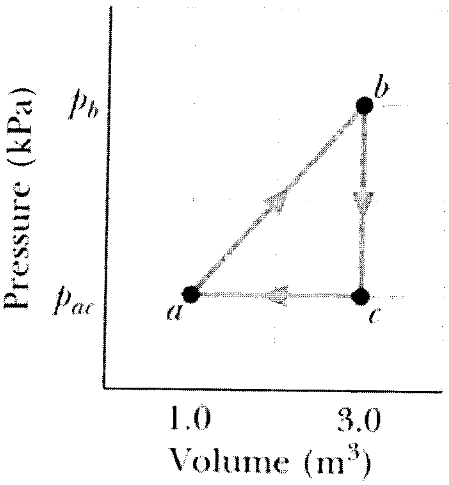
A sample of a monatomic ideal gas is taken through the cyclic process $abca$ shown in the figure. If $p_b = 7.5\ kPa, P_a = P_c = P_{ac} = 2.5\ kPa$ and $T_a = 280K$ find:

a) The number of moles of gas in the sample.

b) The net work done by the gas during one cycle.

c) The net energy added to the gas as heat during one cycle.

d) What is the efficiency of the cycle?



a) $n = \frac{P_a V_a}{R T_a} = \frac{2.5\times10^3\times1}{8.314\times280}$ moles

$n = 1.07$ moles

b) $W_{net} = +A = \frac{2\ m^3\times(7.5-2.5)\times10^3\ \frac{N}{m^2}}{2} = \frac{10^4}{2}J = 5000J$

c) $Q_{net} = W_{net} = \frac{10^4}{2}J = 5000J$

d) $Q_{bc} = nC_v(T_c - T_b) = n\frac{3}{2}R(T_c - T_b) = \frac{3}{2}(P_cV_c - P_bV_b)$

$Q_{bc} = \frac{3}{2}(3\times2.5\times10^3 - 7.5\times3\times10^3)J = -22500J$

$Q_{ca} = nC_p(T_a - T_c) = n\frac{5}{2}R(T_a - T_c) = \frac{5}{2}(P_aV_a - P_cV_c)$

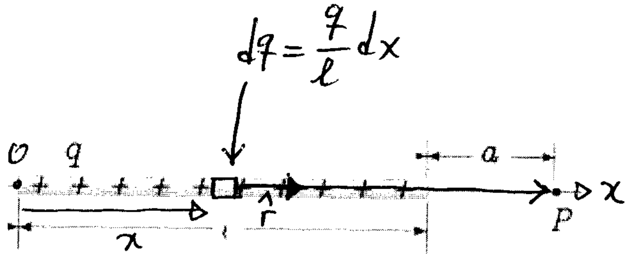
$Q_{ca} = \frac{5}{2}(2.5\times10^3 - 3\times2.5\times10^3)J = -12500J$

$Q_{ab} = Q_{net} - Q_{ca} - Q_{bc} = \frac{10000}{2} + 22500 + 12500J$

$Q_{ab} = 40000J > 0 \Rightarrow Q_H = 40000J$

$W_{net} = 5000J$ $\Rightarrow \varepsilon = \frac{5000}{40000} = 12.5\%$

P4: Electricity: (10 pts)
A non-conducting rod of length ℓ has a total charge q uniformly distributed along its length. Find the electric field vector at point P produced by the rod. (see the figure)



$$\vec{E} = k\int_{bar} \frac{dq}{r^2}\hat{r}$$

$$r = a + \ell - x$$
$$\hat{r} = \hat{i}$$

$$= k\int \frac{\frac{q}{\ell}dx}{(a+\ell-x)^2}\hat{i} = \hat{i}\frac{qk}{\ell}\int_{x=0}^{x=\ell}\frac{dx}{(a+\ell-x)^2} = \hat{i}\frac{qk}{\ell}\left(\frac{-1}{a+\ell-x}\right)_0^\ell$$

$$\vec{E} = \hat{i}\,k\,\frac{q}{\ell}\left(\frac{-1}{a} + \frac{1}{a+\ell}\right) = \hat{i}\,k\,\frac{q}{\ell}\left(\frac{\ell}{a(a+\ell)}\right)$$

Finally $\vec{E} = \hat{i}\,k\,\dfrac{q}{a(a+\ell)}$