Solution

PHYS101 Midterm Exam

Department of Physics

Spring 2017 - April 12, 2017

EASTERN MEDITERRANEAN UNIVERSITY

P1 (7P)	
P2 (12P)	
P3 (11P)	
P4 (8P)	
P5 (2P)	
TOTAL (40P)	

Student Number:	Name and Surname:	Group:	Signature:

Some useful formulae:

DURATION 90 minutes

$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$ $W = \vec{F} \cdot \Delta\vec{r}$ $W_{net} = \Delta K$ $K = \frac{1}{2}mv^2$ $g = 9.80\ \mathrm{m/s^2}$	Constant acceleration kinematic equations: $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$, $\vec{v}(t) = \vec{v}_0 + \vec{a}t$ $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$, $\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$
Newton's second law: $\vec{F}_{net} = \sum_i^N \vec{F}_i = m\vec{a}$	Force of friction $f = F_{fr}$ kinetic: $f_k = \mu_k F_N$, static: $f_s \leq \mu_s F_N$
centripetal (radial) acceleration $a_c = a_r = \frac{v^2}{r}$	average acceleration and average velocity $\vec{a} = \vec{a}_{avg} = \langle\vec{a}\rangle = \frac{\Delta\vec{v}}{\Delta t}$, $\vec{v} = \vec{v}_{avg} = \langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t}$

PROBLEMS:

1) The position of a particle as a function of time is given by $\vec{r} = (5t^2 - 6t + 4)\hat{\imath} + (3t^3 - 8)\hat{\jmath}\ (m)$, where t is in seconds.

a) Determine the particle's instantaneous velocity at $t = 3s$. **(2p)**

$\vec{v}(t) = (10t - 6)\hat{\imath} + (9t^2)\hat{\jmath}$

$\vec{v}(3) = (30 - 6)\hat{\imath} + (9 \times 9)\hat{\jmath} = 24\hat{\imath} + 81\hat{\jmath}\ \mathrm{m/s}$

b) Determine the particle's instantaneous speed at $t = 3s$. **(1p)**

$v(3) = \sqrt{(24)^2 + (81)^2} = 84.5\ \mathrm{m/s}$

c) Determine the direction of particle's velocity ($\theta - coordinate$) at $t = 3s$. **(2p)**

$\theta = \tan^{-1}\left(\frac{81}{24}\right) = 73.5°$ in Region I

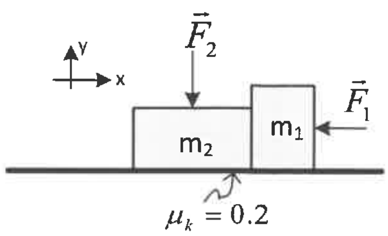
d) Determine the particle's instantaneous acceleration at $t = 3s$. **(2p)**

$\vec{a}(t) = \frac{d\vec{v}}{dt} = 10\hat{\imath} + 18t\hat{\jmath}$

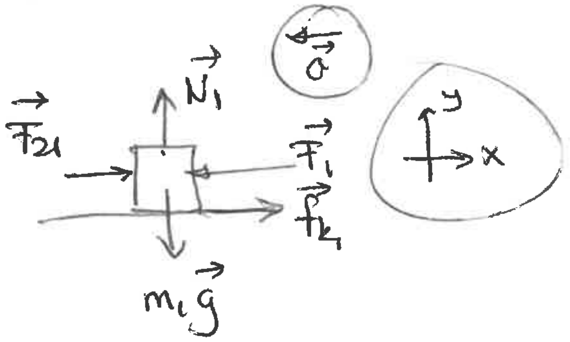
$\vec{a}(3) = 10\hat{\imath} + 54\hat{\jmath}\ (\mathrm{m/s^2})$

2) Two blocks ($m_1 = 4kg$ and $m_2 = 1kg$) on a rough horizontal surface ($\mu_k = 0.2$ for both) are pushed to the left by a horizontal force $F_1 = 73.8N$. Another force $F_2 = 20N$ is vertically pressing the block m_2 to the surface.

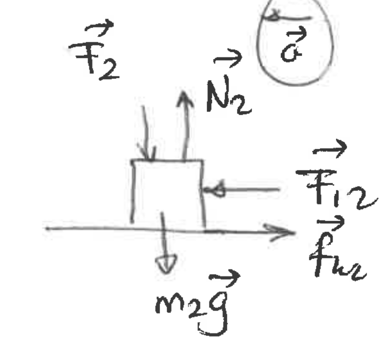
- **a)** Draw the free-body-diagram for each block. **(4p)**
- **b)** Determine the magnitude of the acceleration of the blocks. **(4p)**
- **c)** Find the action and reaction forces exerted by each block on the other: $\vec{F}_{12} =?$ and $\vec{F}_{21} =?$. **(4p)**



FBD-1



FBD-2



From FBD-1 $\Rightarrow \sum \vec{F} = -F_1\hat{i} + f_{k_1}\hat{i} - m_1 g\hat{j} + N_1\hat{j} + F_{21}\hat{i} = -m_1 a\hat{i}$

$F_{21} - F_1 + f_{k_1} = -m_1 a$ --- (1)

$f_{k_1} = \mu_k N_1$

$N_1 - m_1 g = 0$ --- (2)

From FBD-2 $\Rightarrow \sum \vec{F} = -F_2\hat{j} + N_2\hat{j} - m_2 g\hat{j} - F_{12}\hat{i} + f_{k2}\hat{i} = -m_2 a\hat{i}$

$-F_{12} + f_{k2} = -m_2 a$ --- (3)

$f_{k2} = \mu_k N_2$

$N_2 - m_2 g - F_2 = 0$ --- (4)

Adding Eqs (1) and (3), we get

$-F_1 + f_{k1} + f_{k2} = -(m_1+m_2)a \Rightarrow a = \dfrac{F_1 - f_{k1} - f_{k2}}{m_1+m_2}$

According to Eq (2) $N_1 = m_1 g = (4)(9.8) = 39.2N$

Eq (4) $N_2 = m_2 g + F_2 = (1)(9.8) + 20 = 29.8N$

$a = \dfrac{F_1 - \mu_k(N_1+N_2)}{m_1+m_2}$

$a = \dfrac{73.8 - 0.2(39.2+29.8)}{5}$

$a = 12\ m/s^2$

Eq (1) $\Rightarrow F_{21} = F_1 - f_{k1} - m_1 a = 73.8 - 0.2(39.2) - (4)(12)$

$F_{21} \cong 18N$

$\vec{F}_{21} = 18\hat{i}\ N$

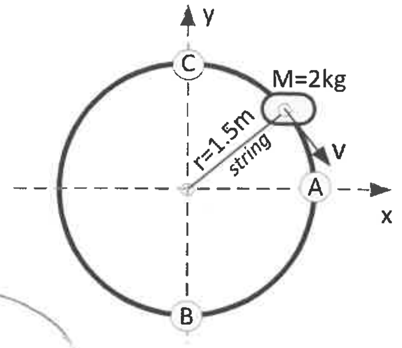
$\vec{F}_{12} = -18\hat{i}\ N$

3) A 2kg rock is attached to a string of negligible mass. It swings in air by following a vertical circle of radius 1.5m. The speeds of the rock for each point are given by $v_A = 7.16m/s,\ v_B = 5.20m/s,$ and $v_C = 3.84m/s$. Ignoring the air resistance and taking $g = 9.81m/s^2$,

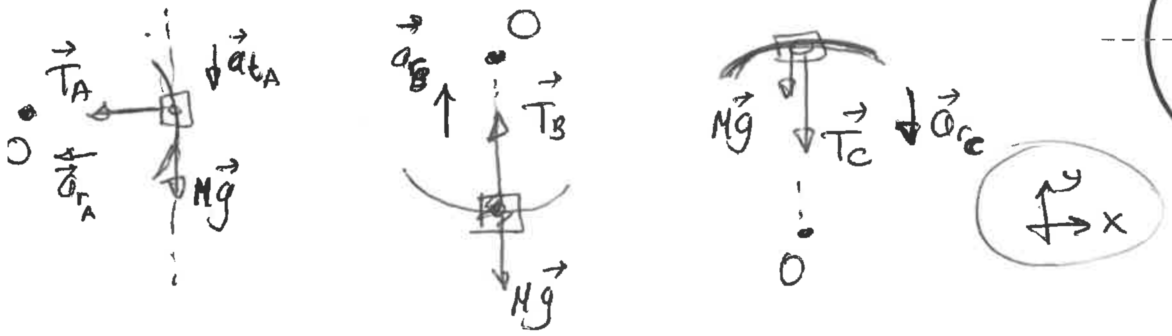
a) Draw the free-body-diagrams for points A, B, and C. **(3p)**

b) Find the magnitude of the total acceleration at point A. **(2p)**

c) Find the tensions in the string: $T_A = ?,\ T_B = ?,$ and $T_C = ?$. **(6p)**



a)



b) $a_{r_A} = \frac{V_A^2}{r} = \frac{(7.16)^2}{1.5} = 34.2\ m/s^2$

$\sum \vec{F_t} = -Mg\hat{j} = -Ma_t\hat{j} \Rightarrow \boxed{a_t = g = 9.81 m/s^2}$

$a_T = \sqrt{a_{r_A}^2 + a_t^2}$

$= \sqrt{(34.2)^2 + (9.81)^2}$

$\boxed{a_T = 35.58\ m/s^2}$

Total Acceleration

c) FBD-A

$\sum \vec{F_x} = -T_A\hat{i} = -Ma_{r_A}\hat{i} \Rightarrow \boxed{T_A = Ma_{r_A} = (2)(34.2) = 68.4N}$

FBD-B

$\sum \vec{F_y} = +T_B\hat{j} - Mg\hat{j} = Ma_{r_B}\hat{j} \Rightarrow T_B - Mg = Ma_{r_B}$

$T_B = Mg + Ma_{r_B} = M(g + a_{r_B})$

$= 2\left(9.81 + \frac{5.2^2}{1.5}\right)$

$\boxed{T_B = 55.67N}$

FBD-C

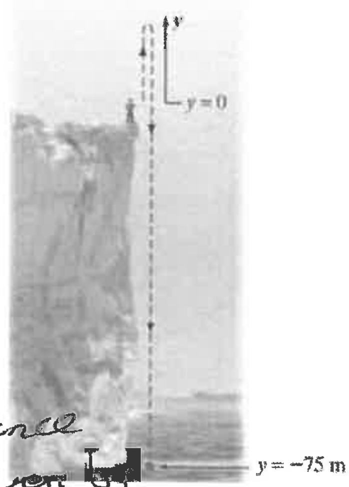
$\sum \vec{F_y} = -Mg\hat{j} - T_C\hat{j} = -Ma_{r_C}\hat{j} \Rightarrow T_C = M(a_{r_C} - g)$

$T_C = 2\left(\frac{3.84^2}{1.5} - 9.81\right)$

$\boxed{T_C = 0.04N}$

4) A stone is thrown vertically upward with a speed of $19.6m/s$ from the edge of a cliff $75m$ high from the sea.

- **a)** Find the time required for the stone to reach its maximum height. **(2p)**
- **b)** What total distance did it travel? **(1p)**
- **c)** How much later does it reach the bottom of the cliff? **(3p)**
- **d)** Find its velocity and speed just before hitting the sea. **(2p)**



a) $\vec{V}_{max} = 0 = \vec{V}_0 + \vec{g} t_{max} = 19.6\hat{j} - 9.8\hat{j} t_{max}$

$t_{max} = 19.6/9.8 = 2s$

b) $\vec{r}_{max} = \vec{r}_0 + \vec{V}_0 t_{max} + \frac{1}{2}\vec{g} t_{max}^2$ ($\vec{r}_0 = 0$)

$= 19.6\hat{j}(2) + \frac{1}{2}(-9.8\hat{j})(2)^2$

$\vec{r}_{max} =$ ~~$29.4\hat{j}$ (m)~~ $19.6\hat{j}$ (m)

Therefore total distance that it traveled is given by

$D =$ ~~$29.4 + 29.4 + 75 = 133.8m$~~ $19.6 + 19.6 + 75 = 114.2m$

c) $\vec{r}_f = \vec{r}_0 + \vec{V}_0 t_f + \frac{1}{2}\vec{g} t_f^2 \Rightarrow -75\hat{j} = 0 + (19.6\hat{j})t_f - 4.9\hat{j} t_f^2$

$-75 = 19.6 t_f - 4.9 t_f^2$

$\Rightarrow 4.9 t_f^2 - 19.6 t_f - 75 = 0$

$t_f = \frac{+19.6 \pm \sqrt{(-19.6)^2 - 4(4.9)(-75)}}{(2)(4.9)}$

$t_f = 6.4s$ ✓

d) $\vec{V}_f = \vec{V}_0 + \vec{g} t_f = 19.6\hat{j} - 9.8\hat{j}(6.4)$

$\vec{V}_f = -43.12\hat{j}$ m/s ✓

Speed $V_f = 43.12$ m/s ✓

5) A force $\vec{F} = 10\hat{\imath} + 9\hat{\jmath} + 12\hat{k}$ (N) acts on a small object of mass $200g$. If the displacement of the object is $\Delta\vec{r} = 5\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ (m),

- **a)** Find the work done by the force $\vec{F}$. **(1p)**
- **b)** If the object's initial speed is $3m/s$, find its final speed. **(1p)**

a) $W = \vec{F} \cdot \Delta\vec{r} = (10\hat{i} + 9\hat{j} + 12\hat{k}) \cdot (5\hat{i} + 4\hat{j} - 2\hat{k})$

$= 50 + 36 - 24 = 62J$

b) $K_f - K_i = W \rightarrow K_f = W + K_i = W + \frac{1}{2} m V_i^2$

$= 62 + \frac{1}{2}(0.2)(3)^2 = 62.9J$

$\frac{1}{2} m V_f^2 = 0.1 V_f^2 = 62.9$

$V_f = \sqrt{629} \cong 25.1$ m/s