Questions:

P1: Thermodynamics: (2+2+3 pts)

10 kg of ice at -20°C is dropped into a big lake at 0°C. (Assume that the lake is so big, so that temperature of the lake remains constant)

a) What is the equilibrium temperature of the system ice+lake?

**Solution:** \( T_e = 0°C = 273K \)

b) How much water of the lake is turned into ice?

**Solution:** The heat absorbed by ice to heat the ice from -20°C to 0°C is

\[
Q_{\text{ice}} = m_{\text{ice}}c_{\text{ice}}(T_f - T_i) = 10\, \text{kg} \cdot 2100\, \text{J/kg/K} \cdot (0 - (-20)) \, K = 4.2 \times 10^5 \, \text{J}.
\]

\( Q_{\text{ice}} \) is the heat extracted from the lake \( \Rightarrow Q_{\text{lake}} = -4.2 \times 10^5 \, \text{J} \). As the lake is at 0°C, this heat extraction leads to freezing a certain amount of ice, which can be calculated as following.

\[
Q_{\text{lake}} = mL_f \Rightarrow m = \frac{Q_{\text{lake}}}{L_f} = \frac{-4.2 \times 10^5 \, \text{J}}{-3.33 \times 10^5 \, \text{J/kg}} = 1.26 \, \text{kg}
\]

c) Calculate the change in entropy for the ice only, for the lake only, and for the isolated system (ice+lake). Does the result agree with the 2nd law of thermodynamics?

**Solution:**

\[
\Delta S_{\text{ice}} = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{m_{\text{ice}}c_{\text{ice}}dT}{T} = m_{\text{ice}}c_{\text{ice}} \ln \left( \frac{T_f}{T_i} \right) = 10\, \text{kg} \cdot 2100\, \text{J/kg/K} \ln \left( \frac{273\, \text{K}}{253\, \text{K}} \right) = 1598\, \text{J/K}
\]

\[
\Delta S_{\text{lake}} = \int \frac{dQ}{T} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = \frac{-4.2 \times 10^5 \, \text{J}}{273\, \text{K}} = -1538\, \text{J/K}
\]

The change of the entropy of the isolated system lake + ice is then:

\[
\Delta S_{\text{lake+ice}} = \Delta S_{\text{ice}} + \Delta S_{\text{lake}} = 1598\, \text{J/K} + 1538\, \text{J/K} = 60\, \text{J/K} > 0
\]

\( \Delta S_{\text{lake+ice}} = 60\, \text{J/K} > 0 \) agrees with the 2nd law of thermodynamics.
P2: Thermodynamics: (2+3+4 pts)

A typical balcony window in Cyprus has an area of 4\text{m}^2, consists either of a single glass pane of thickness 4\text{mm}, or a double glass window of two 4\text{mm} glass panes separated by a 16\text{mm} layer of air. On a regular summer day, the temperature outside is 35\text{°C}, and the air-condition keeps the temperature in the house at convenient 25\text{°C}.

a) Calculate the rate of heat flow through the single glass window.

Solution:

\[ P_{\text{cond}} = A \frac{k_{\text{glass}}}{\ell} \left( T_H - T_C \right) = 4m^2 \cdot 1 \text{W/m} \cdot \text{K} \cdot \frac{(35 - 25) \text{K}}{4 \times 10^{-3} \text{m}} = 1.0 \times 10^4 \text{W} \]

b) Calculate the rate of heat flow through the double glass window with the air in between.

Solution:

\[ P_{\text{cond}} = A \frac{T_H - T_C}{\sum \frac{L_i}{k_i}} A \frac{T_H - T_C}{k_{\text{glass}}} + \frac{L_{\text{air}}}{k_{\text{air}}} + \frac{L_{\text{glass}}}{k_{\text{glass}}} = 4m^2 \cdot \frac{(35 - 25) \text{K}}{4 \times 10^{-3} \text{m}} + \frac{16 \times 10^{-3} \text{m}}{0.026 \text{W/}\text{m} \cdot \text{K}} + \frac{4 \times 10^{-3} \text{m}}{1.0 \text{W/}\text{m} \cdot \text{K}} = 64.2 \text{W} \]

c) How much heat is lost in 2 hours through the single glass window and the three-layer window? Compare the results.

Solution:

\[ Q_1 = P_{\text{cond}} t = 1.0 \times 10^4 \text{W} \cdot 2h \cdot \frac{3600 \text{s}}{1h} = 7.2 \times 10^7 \text{J} \]

\[ Q_1 = P_{\text{cond}} t = 64.2 \text{W} \cdot 2h \cdot \frac{3600 \text{s}}{1h} = 4.62 \times 10^5 \text{J} \]

\[ \frac{Q_1}{Q_2} = \frac{7.2 \times 10^7 \text{J}}{4.62 \times 10^5 \text{J}} = 156 \]

\[ \Rightarrow \text{A single glass window looses} \text{156 times more energy compared to a double glass window of} 4\text{m}^2 \text{in the same time interval.} \]

P3: Thermodynamics: (2+3+3+2 pts)

2 moles of a diatomic ideal gas undergo the following cyclic process, consisting of an isothermal, isobaric, and an adiabatic process. Given \(p_A = 5 \text{atm}, p_B = p_C = 1 \text{atm},\) and \(V_A 2\text{lt}.

a) Calculate the Volumes \(V_B\) and \(V_C\).

Solution:

Process \(A \rightarrow B\) is isothermal \(\Rightarrow p_A V_A = nRT_A = nRT_B = p_B V_B \Rightarrow V_B = \frac{p_A V_A}{p_B} = \frac{5 \text{atm}}{1 \text{atm}} \cdot 2 \text{lt} = 10 \text{lt}.\)

Process \(C \rightarrow A\) adiabatic \(\Rightarrow p_A V_A^{5/3} = p_C V_C^{5/3} \Rightarrow V_C = \left( \frac{p_A}{p_C} \right)^{3/5} V_A = \left( \frac{5 \text{atm}}{1 \text{atm}} \right)^{3/5} 2 \text{lt} = 6.3 \text{lt} \)

b) Calculate the work \(W_{AB}, W_{BC}, W_{CA}\).

Solution:

Process \(A \rightarrow B\) is isothermal, i.e. \(T_A = T_B = \text{const}. \Rightarrow p = \frac{nRT_A}{V} \Rightarrow \frac{p}{V} = \frac{p_A V_A}{V}.\)

\[ W_{AB} = \int_{V_A}^{V_B} p \, dV = \int_{V_A}^{V_B} \frac{p_A V_A}{V} \, dV = p_A V_A \int_{V_A}^{V_B} \frac{dV}{V} = p_A V_A \ln \frac{V_B}{V_A} = 5 \text{atm} \cdot 2 \text{lt} \cdot 101 \frac{J}{\text{atm} \cdot \text{lt}} \ln \frac{10 \text{lt}}{2 \text{lt}} = 1626 \text{J} \]
Process $B \to C$ is isobaric, i.e. $p_B = p_C = \text{const.}$

$$W_{BC} = \int_{V_B}^{V_C} p \, dV = p_B \int_{V_B}^{V_C} dV = p_B (V_C - V_B) = 1\text{atm} \cdot (6.3\text{lt} - 10\text{lt}) \cdot 101 \frac{J}{\text{atm} \cdot \text{lt}} = -373 J$$

Process $C \to A$ is adiabatic.

Way 1:
Because $Q_{CA} = 0$,

$$W_{CA} = -\Delta E_{int,CA} = -nC_V \Delta T_{CA} = -\frac{5}{2} (nRT_A - nRT_C) = -\frac{5}{2} (p_A V_A - p_C V_C) =$$

$$= -\frac{5}{2} (5\text{atm} \cdot 2\text{lt} - 1\text{atm} \cdot 6.3\text{lt}) \cdot 101 \frac{J}{\text{atm} \cdot \text{lt}} = -932 J$$

Way 2:
Because process $C \to A$ is adiabatic. $p_A V_A^{\gamma} = pV^{\gamma} \implies p = \frac{p_A V_A^{\gamma}}{V^{\gamma}}$

$$W_{CA} = \int_{V_C}^{V_A} p \, dV = \int_{V_C}^{V_A} \frac{p_A V_A^{\gamma}}{V^{\gamma}} \, dV = \frac{p_A V_A^{\gamma}}{-\gamma + 1} \left( V_A^{-\gamma + 1} - V_C^{-\gamma + 1} \right) =$$

$$= \frac{1}{-\gamma + 1} \left( \frac{p_A V_A^{\gamma} V_A^{-\gamma + 1} - p_A V_A^{\gamma} V_C^{-\gamma + 1}}{p_C V_C^{\gamma}} \right) = \frac{1}{-\gamma + 1} (p_A V_A - p_C V_C) =$$

$$= \frac{1}{1.4 + 1} (5\text{atm} \cdot 2\text{lt} - 1\text{atm} \cdot 6.3\text{lt}) \cdot 101 \frac{J}{\text{atm} \cdot \text{lt}} = -932 J$$

c) Calculate the Heat $Q_{AB}, Q_{BC}, Q_{CA}$.

Solution:
Process $A \to B$ is isothermal $\Delta E_{int,AB} = 0 = Q_{AB} - W_{AB} \implies Q_{AB} = W_{AB} = 1626 J$.

Process $B \to C$ is isobaric

$$Q_{BC} = nC_P \Delta T_{BC} = \frac{7}{2} nR(T_C - T_B) = \frac{7}{2} (p_C V_C - p_B V_B) =$$

$$= \frac{7}{2} p_B (V_C - V_B) = \frac{7}{2} 1\text{atm} \cdot (6.3\text{lt} - 10\text{lt}) \cdot 101 \frac{J}{\text{atm} \cdot \text{lt}} = -1308 J.$$

Process $C \to A$ is adiabatic $\implies Q_{CA} = 0$

d) Calculate the thermal efficiency efficiency of this cycle.

Solution:
$$e = 1 - \frac{|Q_L|}{Q_H} = 1 - \frac{|Q_{BC} - Q_{CA}|}{Q_{AB}} = 1 - \frac{|-1308 J|}{1626 J} = 0.196 = 19.6\%$$
Given the point charges $q_1 = 5\mu C$ located on the x-axis 4cm from the origin and $q_2 = -2\mu C$ located on the y-axis 3cm from the origin.

a) Calculate the electric field generated by the charge $q_1$ at the position $P$, located at the position $(4i + 3j)$ cm.

Solution:

$$\vec{E}_1 = k_e \frac{q_1}{r_1^2} \hat{r}_1 = 9 \times 10^9 \frac{N}{m^2C^2} \frac{5 \times 10^{-6}C}{(0.03m)^2} \hat{j} = 5 \times 10^7 \frac{N}{C} \hat{j}$$

b) Calculate the electric field generated by the charge $q_2$ at the position $P$.

Solution:

$$\vec{E}_2 = k_e \frac{q_2}{r_2^2} \hat{r}_2 = 9 \times 10^9 \frac{N}{m^2C^2} \frac{-2 \times 10^{-6}C}{(0.04m)^2} \hat{i} = -1.125 \times 10^7 \frac{N}{C} \hat{i}$$

c) Calculate the total electric field at the point $P$.

Solution:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 5 \times 10^7 \frac{N}{C} \hat{j} - 1.125 \times 10^7 \frac{N}{C} \hat{i} = (-1.125\hat{i} + 5\hat{j}) \times 10^7 \frac{N}{C}$$

d) Calculate the force exerted by the charges $q_1$ and $q_2$ on a test charge $q_3 = -3\mu C$ located at $P$.

Solution:

$$\vec{F}_3 = q_3 \vec{E} = -3 \times 10^{-6}C(-1.125\hat{i} + 5\hat{j}) \times 10^7 \frac{N}{C} = (-33.75\hat{i} + 150\hat{j}) \frac{N}{C}$$
P5: Electricity: (7 pts)

Calculate the electric field of a uniformly charged ring with a total charge \( Q \) and radius \( a \), at a point \( P \) on the \( x \)-axis at a distance \( x \) from the centre of the ring.

**Solution:**

\[
r = \sqrt{x^2 + a^2}, \quad \hat{r} = \frac{x\hat{i} - a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}}
\]

\[
d\vec{E} = k \frac{dq}{r^2} \hat{r} = k \frac{\lambda a d\theta}{x^2 + a^2} \frac{x\hat{i} - a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}}
\]

integrating over \( \theta \) gives:

\[
\vec{E} = k \int_0^{2\pi} \left( \frac{\lambda a}{x^2 + a^2} \frac{x\hat{i} - a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}} \right) d\theta =
\]

\[
= k \int_0^{2\pi} \left( \frac{\lambda a}{x^2 + a^2} \frac{x\hat{i} - a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}} \right) d\theta + k \int_0^{2\pi} \left( \frac{\lambda a}{x^2 + a^2} \frac{-a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}} \right) d\theta + k \int_0^{2\pi} \left( \frac{\lambda a}{x^2 + a^2} \frac{x\hat{i} - a \cos \theta \hat{j} - a \sin \theta \hat{k}}{\sqrt{x^2 + a^2}} \right) d\theta =
\]

\[
= k \left( \frac{\lambda a x}{(x^2 + a^2)^{3/2}} \hat{i} - \frac{\lambda a^2}{(x^2 + a^2)^{3/2}} \hat{j} - \frac{\lambda a^2}{(x^2 + a^2)^{3/2}} \hat{k} \right)
\]

\[
= k \frac{\lambda a 2\pi x}{(x^2 + a^2)^{3/2}} \hat{i} = kQ x \frac{x}{(x^2 + a^2)^{3/2}}
\]

of course you can argue directly that the component of the electric field in \( y \) and \( z \) direction cancel, as the calculation shows, and say

\[
dE_x = k \frac{dq}{r^2} = k \frac{\lambda a d\theta}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}}
\]

as there is no \( \theta \)-dependence of \( x \) and \( a \), the integral over \( \theta \) gives \( 2\pi \), resulting

\[
E_x = k \frac{\lambda a 2\pi x}{(x^2 + a^2)^{3/2}} = kQ x \frac{x}{(x^2 + a^2)^{3/2}}.
\]