# Classification of EM Problems

The classification of the EM problem is important in helping to find the best method for solution.

1) The solution region of the problem,
2) Equations describing the problem,
3) Boundary Conditions.

## *1) The Solution Regions*

Solution regions may be classified as, closed region, bounded, or open region, unbounded. i.e. Wave propagation in a waveguide is a closed region problem, where radiation from a dipole antenna is an open region problem.

A problem is also classified in terms of the electrical, constitutive properties. We shall be concerned with simple materials here.

## *2) Classification of the Equations Describing the Problem*

EM problems may be stated by using:

1) Differential Equations
2) Integral Equations
3) Integro-Differential Equations

In general, EM problems have the following form:

$$L\Phi = g$$

$L$ : Operator (integral, differential, integro-differential),
$g$ : Excitation or source,
$\Phi$ : Unknown function.

Example: The Poisson's Equation in the differential form (differential operator).

$$-\nabla^2 V = \frac{\rho_v}{\varepsilon}$$

Here,

$$L = -\nabla^2$$

$$g = \frac{\rho_v}{\varepsilon}$$

$$\Phi = V$$

Example: The Poisson's Equation in the integral form (integral operator).

$$V(\bar{r}) = \int_v \frac{\rho_v(\bar{r}')}{4\pi\varepsilon_o |\bar{r} - \bar{r}'|} dv'$$

Where

$$L = \int_v \frac{dv'}{4\pi |\bar{r} - \bar{r}'|}$$

$$g = V(\bar{r})$$

$$\Phi = \frac{\rho_v(\bar{r}')}{\varepsilon_o}$$

This formulation is used in the MoM to find the charges on conductors when the potential distribution is known.

Example: Electric field intensity of a wire antenna integro-differential operator).

$$E_z = \frac{1}{j\omega\varepsilon_o}\int_{-\ell/2}^{\ell/2}\left[\frac{\partial^2 G(z,z')}{\partial z^2} + k^2 G(z,z')\right] I(z')\,dz'$$

Where, $G(z,z') = \dfrac{e^{-jkr}}{4\pi r}$.

EM problems satisfy second order partial differential equations (PDE).

i.e. Wave equation, Laplace's equation.

A two dimensional second order PDE may have the following form:

$$a\frac{\partial^2\Phi}{\partial x^2} + b\frac{\partial^2\Phi}{\partial x\partial y} + c\frac{\partial^2\Phi}{\partial y^2} + d\frac{\partial\Phi}{\partial x} + e\frac{\partial\Phi}{\partial y} + f\Phi = g$$

The operator is:

$$L = a\frac{\partial^2}{\partial x^2} + b\frac{\partial^2}{\partial x\partial y} + c\frac{\partial^2}{\partial y^2} + d\frac{\partial}{\partial x} + e\frac{\partial}{\partial y} + f$$

Any linear second order PDE can be classified as follows:

$$If,\ b^2 - 4ac < 0 \quad Elliptic$$
$$If,\ b^2 - 4ac = 0 \quad Parabolic$$
$$If,\ b^2 - 4ac > 0 \quad Hyperbolic$$

The differential equation is said to be **non-linear** if the coefficients (a, b, c, d, e, f) is a function of $x$ and $y$ .

If, $g(x,y)=0$, then PDE is homogeneous.
If, $g(x,y)\neq 0$, then PDE is inhomogeneous.

Examples: Elliptic PDE, Poisson's and Laplace's Equations

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=g(x,y) \quad Poisson's\ Eqn.$$
$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0 \qquad Laplace's\ Eqn$$

For both cases a=c=1, b=0.

$$b^2 - 4ac < 0$$

An elliptic PDE usually models the closed region problems.

Example: Hyperbolic PDE’s, the Wave Equation in one dimension.

$$\frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{u^2}\frac{\partial^2 \Phi}{\partial t^2} = 0$$

A Hyperbolic PDE usually models the Propagation Problems (Open region problems).

$$a = u^2, b = 0, c = -1$$
$$b^2 - 4ac > 0$$

Example: Parabolic PDE, Diffusion Equation for an RC transmission line.

$$\frac{\partial^2 V}{\partial x^2} - RC\frac{\partial V}{\partial t} = 0$$
$$-\frac{1}{RC}\frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial t} = 0$$

$$a = -\frac{1}{RC}, \ b = c = 0$$

$$b^2 - 4ac = 0$$

A Parabolic PDE usually arises in propagation problems.

### *3) Classification of Boundary Conditions:*

Consider again,

$$L\Phi = g$$

The solution of this equation is to find the unknown function $\Phi$, within a solution region $R$, subject to certain boundary conditions on surface $S$ bounding $R$.

These boundary conditions are Dirichlet and Neumann type boundary conditions.

1) Dirichlet B.C.

$$\Phi(r) = 0, \; R \; on \; S$$

i.e $\Phi$, vanishes on $S$

2) Neumann B.C

$$\frac{\partial \Phi(r)}{\partial n} = 0, \; R \; on \; S.$$

i.e. the normal derivative of $\Phi$ vanishes on S.

3) Mixed Boundary Conditions

**A PDE in general can have both:**

1) Boundary Values
2) Initial values

Example:

Consider the 1D wave equation:

$$\frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{u^2}\frac{\partial^2 \Phi}{\partial t^2} = 0$$

The solution of this second-order wave equation requires the following initial conditions:

$$\Phi\left(x,0\right) = \varphi(x)$$

$$\left.\left(\frac{\partial \Phi\left(x,t\right)}{\partial t}\right)\right|_{t=0} = \theta(x)$$