Faculty of Engineering

DEPARTMENT of ELECTRICAL AND ELECTRONIC ENGINEERING

EENG (INFE)115 Introduction to Logic Design

Instructors:

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Midterm EXAMINATION

November 18, 2017

Duration : 100 minutes

Number of Questions: 4

Good Luck

STUDENT'S	
NUMBER	
NAME	
SURNAME	**SOLUTIONS**
GROUP NO	

Question		Points
1		25
2		25
3		25
4		25
TOTAL		100

Read the following instructions carefully:

1. **Calculators** are not allowed.
2. Switch off **mobile phones** and **do not borrow** any stationery from your friends.
3. In your solutions, **show all details** you claim credit for.

Question 1

a) Convert the following number with the indicated base to decimal. (Show your steps and only the first two digits after the decimal point.) (**5 pts**.)

$$(245.34)_6 = 2\times6^2+4\times6^1+5\times6^0+3\times6^{-1}+4\times6^{-2}$$

=

$$= 72+24+5+\frac{3}{6}+\frac{4}{36}=101+\frac{22}{36}=101.61$$

b)Convert the following decimal numbers to numbers in base 5. (Show your steps.) (**6 pts**.)

Integer	Remainder
48	
9	3
1	4
0	1

i.

$$(48)_{10}=(143)_5$$

ii. 0.88

.

	Integer		Fraction	Coefficient
0.88 x 5=	4	+	0.4	$a_{-1}=4$
0.4 x 5 =	2	+	0.0	$a_{-2}=2$

$$(0.88)_{10}=(0.42)_5$$

c) Convert $(1110111110.01101)_2$ to hexadecimal and to octal. (**4 pts**.)

001110111110.01101000

3 B E 6 8

$$(001110111110.01101000)_2 = (3BE.68)_{16}$$

$$\left(001110111110.011010\right)_2 = (1676.32)_8$$

1 6 7 6 3 2

d) Convert +31 and +56 to binary, using the **signed-1's complement** representation and **enough digit to accommodate** the numbers. Then perform the binary equivalent of (-31) +(+56) and (-31)-(+56). (**10 pts**.)

$(31)_{10} = (11111)_2$

$(56)_{10} = (111000)_2$

$(31)_{10} + (56)_{10} = (87)_{10} = (1010111)_2$

Therefore

$(+87)_{10} = (01010111)_2 \Rightarrow$ requires 8 bits.

$(+31)_{10} = (00011111)_2$ signed-1's complement of $(-31)_{10} = (11100000)$

$(+56)_{10} = (00111000)_2$ signed-1's complement of $(-56)_{10} = (11000111)_2$

(- 3 1)		1 1 1 0 0 0 0 0
+ (+ 5 6)	+	0 0 1 1 1 0 0 0
(+ 2 5)		1 0 0 0 1 1 0 0 0
	+	1
		0 0 0 1 1 0 0 1

(-31)-(+56) = (-31)+(-56)

(- 3 1)		1 1 1 0 0 0 0 0
+ (- 5 6)	+	1 1 0 0 0 1 1 1
(- 8 7)		1 1 0 1 0 0 1 1 1
	+	1
		1 0 1 0 1 0 0 0

Since the most significant bit is 1 that is a negative number. In order to find the result -(1's complement of the result)= -(01010111) = -87_{10}

Question 2

a) Simplify the following Boolean functions to a minimum number of literals using **algebraic manipulation**.

$$F(x,y,z) = x'y'z + xy' + yz + xyz'$$

i. $$F(x,y,z) = y'(x'z + x) + y(z + xz') = y'\underbrace{(x' + x)}_{1}(z + x) + y\underbrace{(z + z')}_{1}(z + x)$$

$$F(x,y,z) = \underbrace{(y' + y)}_{1}(x + z) = (x + z)$$

$$F(w,x,y,z) = \sum(0,1,2,4,6,8,10)$$

$$F(w,x,y,z) = w'x'y'z' + w'x'y'z' + w'x'y'z + w'x'yz' + w'xy'z' + w'xyz' + wx'y'z' + wx'yz'$$

$$F(w,x,y,z) = w'x'y'\underbrace{(z' + z)}_{1} + w'x'\underbrace{(y' + y)}_{1}z' + w'xz'\underbrace{(y + y')}_{1} + wx'\underbrace{(y' + y)}_{1}z'$$

b)

$$F(w,x,y,z) = w'x'y' + w'x'z' + w'xz' + wx'z' + w'x'z'$$

$$F(w,x,y,z) = w'x'y' + w'z'\underbrace{(x' + x)}_{1} + \underbrace{(w + w')}_{1}x'z'$$

$$F(w,x,y,z) = w'x'y' + w'z' + x'z'$$

(15 pts.)

b) Express the following function as a sum of minterms and as a product of maxterms (**10 pts**.)

$$F(w,x,y,z) = w'(x \oplus y) + w(y \odot z)$$
$$F(w,x,y,z) = w'(x'y + xy') + w(y'z' + yz) = w'x'y + w'xy' + wy'z' + wyz$$
$$F(w,x,y,z) = w'x'y(z' + z) + w'xy'(z' + z) + w(x' + x)y'z' + w(x' + x)yz$$
$$F(w,x,y,z) = w'x'yz' + w'x'yz + w'xy'z' + w'xy'z + wx'y'z' + wxy'z' + wx'yz + wxyz$$
$$F(w,x,y,z) = m_2 + m_3 + m_4 + m_5 + m_8 + m_{12} + m_{11} + m_{15}$$
$$F(w,x,y,z) = \sum(2,3,4,5,8,11,12,15)$$

and Product of Maxterms:

$$F(w,x,y,z) = \prod(0,1,6,7,9,10,13,14)$$

Question 3

For a given Boolean function

$$F(A,B,C,D) = \sum (2, 4, 7, 13, 14)$$

$$d(A,B,C,D) = \sum ((A \oplus B \oplus C \oplus D)')$$

a) Determine the sum of products (SOP).
b) Implement F with only NAND gates.
c) Determine the product of sums (POS).
d) Implement F with only NOR gates.

(**25 pts.**)

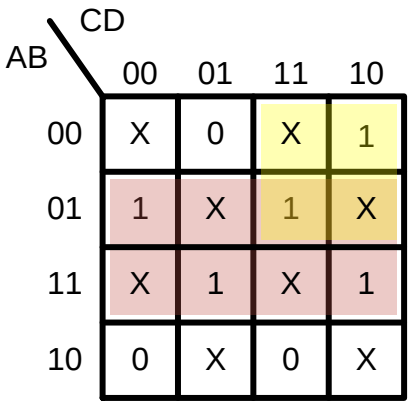
For a given Boolean function

$$F(A,B,C,D) = \sum (2, 4, 7, 13, 14)$$

$$d(A,B,C,D) = \sum ((A \oplus B \oplus C \oplus D)')$$

a. **Determine the sum of products (SOP).**

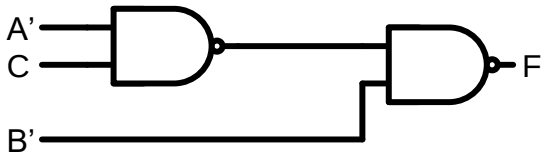
A	B	C	D	d
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1



F = B + A'C

b. Implement *F* with only NAND gates.

F = B + A'C
F = (B'.(A'C)')'



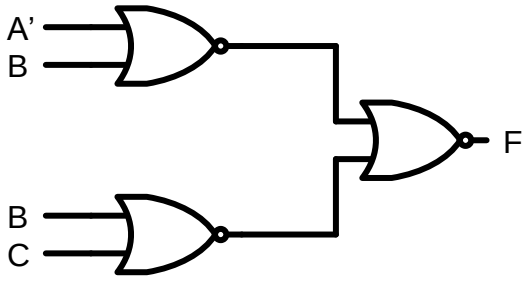
c. Determine the product of sums (POS).

AB \ CD	00	01	11	10
00	X	0	X	1
01	1	X	1	X
11	X	1	X	1
10	0	X	0	X

F’ = AB’ + B’C’
F = (A’+B) (B+C)

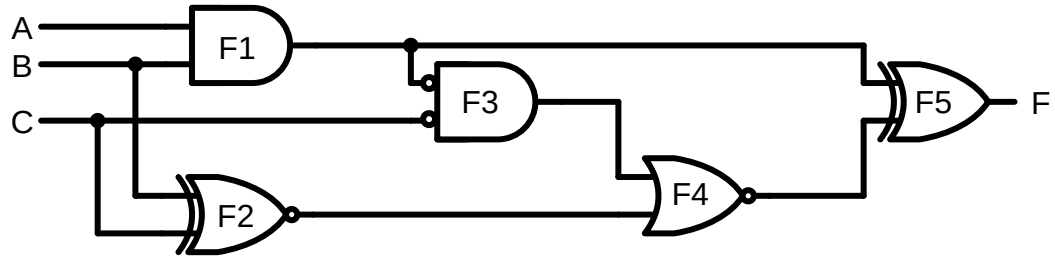
d. Implement *F* with only NOR gates.

F = (A’+B) (B+C)
F = ((A’+B)’+(B+C)’)’



Question 4

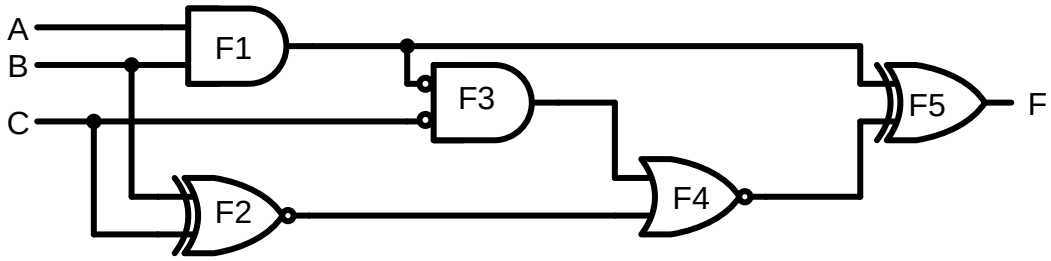
Analyse the following circuit



a) Use the truth table and determine the output F as sum of minterms and product of maxterms.
b) Use Karnaugh Map to simplify the equation obtained in (a).
c) Implement the SOP and POS circuits with minimum number of gates.

(**25 pts.**)

Analyse the following circuit



F1 = A.B, F2 = (B⊕C)',
F3 = (A.B)' . C' = (A'+B') C' = A'C'+B'C'
F4 = ((B⊕C)' + A'C'+B'C')'
= (((B⊕C)')'.(A'C'+B'C')')
= (B⊕C) (A+C)(B+C)
= (BC'+ B'C) (AB+AC+BC+C)
= ABBC'+~~ABB'C+ACBC'~~+ACB'C+~~BCBC'~~+~~BCB'C+CBC'~~+CB'C
= ABC' + AB'C + B'C
= ABC' + B'C (A+1)
= ABC' + B'C
F5 = (ABC' + B'C') ⊕ A.B

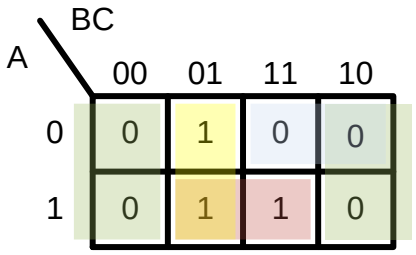
a. **Use the** truth table **and determine the output** F **as** sum of minterms **and** product of max terms**.**

A	B	C	A'	B'	C'	A.B	ABC'	B'C	F4	F5	SOP	POS
0	**0**	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**		**0**
0	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**1**	**1**	**1**	**1**	
0	**1**	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**0**	**0**		**2**
0	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**0**		**3**
1	**0**	**0**	**0**	**1**	**1**	**0**	**0**	**0**	**0**	**0**		**4**
1	**0**	**1**	**0**	**1**	**0**	**0**	**0**	**1**	**1**	**1**	**5**	
1	**1**	**0**	**0**	**0**	**1**	**1**	**1**	**0**	**1**	**0**		**6**
1	**1**	**1**	**0**	**0**	**0**	**1**	**0**	**0**	**0**	**1**	**7**	

$$F(A,B,C,D) = \sum (1, 5, 7)$$

$$F(A,B,C,D) = \prod (0, 2, 3, 4, 6)$$

b. Use Karnaugh Map **to simplify the equations obtained in** (a).



F = B'C + AC

F' = C + A'B

F = C' (A+B')

c. Implement the circuits with minimum number of gates.