PLANE WAVES IN A CONDUCTING MEDIUM

II. GOOD CONDUCTING CASE

If $\varepsilon" >> \varepsilon'$ or $\left(\frac{\sigma}{w\varepsilon} >> 1\right)$, conduction current dominates considerably over the displacement current. Then,

$$\alpha = w\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} - 1\right]^{1/2} \approx w\sqrt{\frac{\varepsilon\mu}{2}}\left[\left(\frac{\sigma}{w\varepsilon}\right) - 1\right]^{1/2}$$

$$\alpha \approx w\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\frac{\sigma}{w\varepsilon}} = \sqrt{\frac{w\mu\sigma}{2}} = \sqrt{\frac{2\pi f\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

Then, $\alpha \approx \sqrt{\pi f\mu\sigma}$, α is a function of frequency.

$$\beta = w\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{w\varepsilon}\right)^2} + 1\right]^{1/2} \approx w\sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma}{w\varepsilon} + 1\right]^{1/2}$$

$$\beta = w\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{\frac{\sigma}{w\varepsilon}} = \sqrt{\pi f\mu\sigma}$$

$\alpha = \beta = \sqrt{\pi f\mu\sigma}$. Then,

$$\gamma = \alpha + j\beta = \sqrt{\pi f\mu\sigma}\,(1+j)$$

<u>Intrinsic Impedance:</u>

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon - j\frac{\sigma}{w}}} = \sqrt{\frac{\mu}{\varepsilon}}\left(1 - j\frac{\sigma}{w\varepsilon}\right)^{-1/2}$$

$$\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}\left(-j\frac{\sigma}{w\varepsilon}\right)^{-1/2} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{-j\frac{\sigma}{w\varepsilon}}} = \sqrt{\frac{\mu w}{\sigma}}\sqrt{j} = \sqrt{\frac{\mu w}{\sigma}}\frac{(1+j)}{\sqrt{2}}$$

$$\eta_c \approx (1+j)\sqrt{\frac{w\mu}{2\sigma}}$$

$$\sqrt{\frac{w\mu}{2\sigma}} = \sqrt{\frac{w\mu\sigma}{2\sigma^2}} = \sqrt{\frac{w\mu\sigma}{2}}\frac{1}{\sigma} = \frac{\alpha}{\sigma}$$

So, $\eta_c = (1+j)\frac{\alpha}{\sigma} = \frac{\alpha\sqrt{2}}{\sigma}\angle 45^0$

The results show that $\bar{H}$ lags $\bar{E}$ by 45^0 in time.

Phase Velocity

$$u_p = \frac{w}{\beta} = \frac{w}{\sqrt{\frac{w\mu\sigma}{2}}} = \sqrt{\frac{2w}{\mu\sigma}}$$

The wavelength,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\pi f \mu\sigma}} = 2\sqrt{\frac{\pi}{f\mu\sigma}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\alpha} = 2\pi\left(\frac{1}{\alpha}\right) = 2\pi\delta$$

Example:

Consider copper for which $\sigma = 5.80X10^7 (S/m)$ $\varepsilon = \varepsilon_0$, $\mu \simeq \mu_0$. Take $f = 3MHz$.

Solution:

Let us first find the wavelength of a wave in free space at this frequency:

$$\lambda_0 = \frac{c}{f} = \frac{3X10^8}{3X10^6} = 100\,meters$$

Calculate the loss tangent:

$$\frac{\sigma}{\omega\varepsilon} = \frac{5.8X10^7}{2\pi X 3X10^6 X \frac{1}{36\pi} X10^{-9}} = 3.48X10^{11} >> 1$$

Copper behaves like a good conductor at $f = 3MHz$.

The wavelength in copper will then be:

$$\lambda = 2\left(\frac{\pi}{f\,\mu\sigma}\right)^{1/2} = 2\left(\frac{\pi}{3X10^6\,X\,4\pi X10^{-7}\,X\,5.8X10^7}\right)^{1/2} = 0.239X10^{-3}\,m\ .$$

or

$\lambda = 0.239mm$ (compare with 100 meters in air!)

$$\alpha = \sqrt{\pi\, f\,\mu\sigma} = \left(\pi\,X\,3X10^6\,X\,4\pi X10^{-7}\,X\,5.8X10^7\right)^{1/2} = 2.62X10^4\,(Np/m)$$

The depth of penetration:

$$\delta = \frac{1}{\alpha} = 0.38X10^{-4}\,m = 0.038mm\ .$$

δ at $f = 10GHz$ is $0.66\mu m$.

The frequency at which $\sigma = w\varepsilon$ is $f = \dfrac{5.8X10^7}{2\pi\varepsilon_0} = 1.04X10^{18}\,Hz$

Thus at frequencies of even several gigahertz, copper behaves like an excellent conductor. The material is actually a good conductor well into the ultraviolet region. The skin-depth at optical frequencies ($w \sim 10^{15}$) is roughly $10^{-8}m$, which explains why metals are opaque.

<u>Example:</u>

Consider sea-water, for which $\sigma = 4\,mhos/m$, $\varepsilon = 80\varepsilon_0$, $\mu = \mu_0$. Take $f = 25kHz$.

<u>Solution:</u>

$$\frac{\sigma}{w\varepsilon} = \frac{4}{2\pi\,X\,25X10^3\,X\,80X\,\dfrac{1}{36\pi}\,X10^{-9}} = 36000 >> 1$$

Sea-water is good conductor at $f = 25kHz$.

$$\lambda_{air} = \frac{3X10^8}{25X10^3} = 12km$$

$$\lambda_{sea-water} = 2\left(\frac{\pi}{f\,\mu\sigma}\right)^{1/2} = 2\left(\frac{\pi}{25X10^3\,X\,4\pi X10^{-7}\,X\,4}\right)^{1/2} = \frac{1}{4\pi X10^{-2}}\left(\frac{1}{f_{kHz}}\right)^{1/2}$$ or

$$\delta = \frac{8}{\sqrt{f_{kHz}}}\,(meter).$$

$f\,(kHz)$	$\delta(m)$
10	2.531
15	2.067
20	1.789
25	1.6
30	1.462
35	1.353
100	0.8