OVERVIEW OF ELECTROMAGNETIC THEORY

Time-Varying Fields

Electromagnetic Field and Source Quantities

Time varying electromagnetic fields (which cause radiation by an antenna) are related to each other by the following differential equations:

Curl Equations:

$$\nabla X\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$ (Faraday's Induction Law)

$$\nabla X\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$ (Ampere's Circuital Law)

We can see from these curl equations that time-varying electric fields create magnetic field and vice versa.

Divergence Equations:

$$\nabla.\bar{D} = \rho_v \ (\text{Gauss Law for Electric Fields})$$

$$\nabla.\bar{B} = 0 \ (\text{Gauss Law for Magnetic Fields})$$

Definitions:

Field Terms

$\bar{E}$ **: Electric Field Intensity**	(V/m)
$\bar{D}$ **: Displacement Vector**	(C/m^2)
$\bar{H}$ **: Magnetic Field Intensity**	(A/m)
$\bar{B}$ **: Magnetic Flux density**	$(T, Wb/m^2)$

Source Terms

ρ_v **: Electric charge density**	(C/m^3)
$\bar{J}$ **: Electric Current Density**	(A/m^2)

The Maxwell's equations together with the Continuity Equation;

$$\nabla.\bar{J} + \frac{\partial \rho}{\partial t} = 0$$

and the Lorentz Force Equation:

$$\bar{F} = q\left(\bar{E} + \bar{u} X \bar{B}\right)\ (Newton)$$

form the foundation of the electromagnetic theory.

Integral Forms of the Maxwell's Equations

Faraday's Law of Electromagnetic Induction:

$$v_{emf} = \oint_C \bar{E}.\bar{dl} = -\int_s \frac{\partial \bar{B}}{\partial t}.\hat{n}ds$$

Ampere's Law:

$$\oint_C \bar{H}.\bar{dl} = \int_s (\bar{J} + \frac{\partial \bar{D}}{\partial t}).\hat{n}ds$$

Gauss Law for Electric Fields:

$$\oint_s \bar{D}.\hat{n}ds = \iiint_v \rho_v dv = Q_{equ}$$

Gauss Law for Magnetic Fields:

$$\oint_s \bar{B}.\hat{n}ds = 0$$

Constitutive Relations

In free space, the flux densities and field intensities are related by the constitutive relations:

$$\bar{D} = \varepsilon_0 \bar{E}$$
$$\bar{B} = \mu_0 \bar{H}$$

Where $\varepsilon_0 \approx 8.854 X 10^{-12} (F/m)$ is the permittivity and $\mu_0 = 4\pi X 10^{-7} (H/m)$ the permeability of free space.

In a material medium,

$$\bar{D} = \varepsilon \bar{E}$$
$$\bar{B} = \mu \bar{H}$$

with

$$\varepsilon = \varepsilon_r \varepsilon_0$$
$$\mu = \mu_r \mu_0$$

Where ε_r and μ_r are the relative permittivity and permeability of the medium respectively. In an inhomogeneous medium permittivity and permeability are functions of position. So,

$$\bar{D} = \varepsilon(x, y, z) \bar{E}$$
$$\bar{B} = \mu(x, y, z) \bar{H}$$

In a conducting material, the conduction current density is:

$$\bar{J} = \sigma \bar{E}$$

where, σ (S/m) is the conductivity of the material.

Gradient, Divergence, Curl

The source and field quantities in Maxwell's equations depend on position and are represented by vector fields. In Cartesian coordinates:

$$\bar{A} = A_x(x, y, z)\hat{a}_x + A_y(x, y, z)\hat{a}_y + A_z(x, y, z)\hat{a}_z$$

The spatial derivatives of vector fields appear in the Maxell's equations such as gradient, divergence and curl. In the rectangular coordinate system, the **del** operator is:

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

The gradient of a scalar field $f(x, y, z)$ is the vector field:

$$\nabla f = \frac{\partial f}{\partial x}\hat{a}_x + \frac{\partial f}{\partial y}\hat{a}_y + \frac{\partial f}{\partial z}\hat{a}_z$$

The divergence of a vector field is a scalar function:

$$\nabla.\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

The curl of a vector field is another vector evaluated as:

$$\nabla X\bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

<u>Laplacian</u>

The Laplacian operator is:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

When applied to a vector field $\bar{A}$:

$$\nabla^2\bar{A} = -\nabla X\nabla\bar{A} + \nabla\left(\nabla.\bar{A}\right)$$

When applied to a scalar field f :

$$\nabla^2 f = \nabla.\nabla f$$

Wave Propagation

EM fields are waves that propagate in free space. Therefore, we will review the basic wave equations and their solutions.

Maxwell's Equations are first-order coupled equations. But, they can be written as single second-order partial differential equations. Consider a linear, isotropic and homogeneous dielectric (simple medium) medium. The curl of both sides of the Faraday's Law equation is:

$$\nabla X \nabla X \bar{E} = -\mu \frac{\partial}{\partial t} \nabla X \bar{H}$$

Substitute the Ampere's Law equation in the above equation considering source free ($\bar{J} = \bar{J}_{imp} = 0$), non-conducting region $\bar{J} = \sigma \bar{E} = 0$:

$$\nabla X \bar{H} = \varepsilon \frac{\partial \bar{E}}{\partial t}$$

Results,

$$\nabla X \nabla X \bar{E} = -\mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$
$$\nabla X \nabla X \bar{E} = \nabla\left(\nabla . \bar{E}\right) - \nabla^2 \bar{E}$$

In the source free region $\nabla . \bar{E} = \rho = 0$:

$$\nabla^2 \bar{E} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

In the Cartesian coordinate system:

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} - \mu\varepsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

Denote the wave velocity, $u = \frac{1}{\sqrt{\varepsilon\mu}} \left(m/s\right)$,

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} - \frac{1}{u^2} \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

In free space $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3X10^8 (m/s)$

$$\frac{\partial^2 \bar{E}}{\partial x^2} + \frac{\partial^2 \bar{E}}{\partial y^2} + \frac{\partial^2 \bar{E}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

This equation is valid for any scalar component of the electric field intensity E_x, E_y and E_z. i.e. for the E_x component:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial E_x}{\partial t^2} = 0$$

For a special case where $\bar{E} = \hat{a}_x E_x(z)$,

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2} = 0$$

The solution of the above wave equation is:

$$E(z,t) = E^+(z - ct) + E^-(z + ct)$$

Where E^+ represents the wave travelling in the *+z* direction and E^- represents the wave travelling in the *–z* direction.

Boundary Conditions

In order to solve electromagnetic problems including more than one material with different constitutive parameters, it is necessary to know the boundary conditions that the field vectors ($\bar{E}, \bar{D}, \bar{H}, \bar{B}$) satisfy at an interface. These conditions are derived by using the integral forms of the Maxwell's equations across the interface of two materials.

Boundary condition for the tangential components of $\bar{E}$:

$$\hat{n} \mathrm{X} \left(\bar{E}_2 - \bar{E}_1 \right) = 0 \quad \text{or} \quad E_{t_2} = E_{t_1}$$

Boundary condition for the tangential components of $\bar{H}$:

$$\hat{n}\text{X}\left(\bar{H}_2 - \bar{H}_1\right) = \bar{J}_s \quad \text{or} \quad H_{t_2} - H_{t_1} = \bar{J}_s$$

Boundary condition for the normal component of $\bar{D}$:

$$\hat{n}.\left(\bar{D}_2 - \bar{D}_1\right) = \rho_s \quad \text{or} \quad D_{n_2} - D_{n_1} = \rho_s$$

Boundary condition for the normal component of $\bar{B}$:

$$\hat{n}.\left(\bar{B}_2 - \bar{B}_1\right) = 0 \quad \text{or} \quad B_{n_2} = B_{n_1}$$

The subscripts *1* and *2* are used to represent the fields in material *1* and material *2*. $\hat{n}$ is a unit vector perpendicular to the interface pointing into the second material. $\bar{J}_s$ is the electric surface current density in $\left(A/m\right)$ and ρ_s is the surface charge density in $\left(C/m^3\right)$.

Good conductors can be approximated as perfect electric conductors (PEC). Inside a PEC, all time-varying fields are zero.

The boundary conditions for the PEC's are reduced to:

$$\hat{n}\text{X}\,\bar{E} = 0 \quad \text{or} \quad E_t = 0$$

$$\hat{n}\text{X}\bar{H} = \bar{J}_s \quad \text{or} \quad H_t = \bar{J}_s$$

$$\hat{n}.\bar{D} = \rho_s$$

$$\hat{n}.\bar{B} = 0$$

Where $\hat{n}$ is normal to the PEC.

Time and Frequency Domain Representations of Maxwell's Equations

Since the sources are time varying, due to the linearity of the Maxwell's Equations, the field quantities also are time-varying. It is known that, time harmonic quantities are represented by phasors.

The phasor expressions of the Maxwell's equations are obtained from the time-varying equations by replacing:

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\nabla \mathrm{X} \bar{E} = -j\omega\mu \bar{H}$$
$$\nabla \mathrm{X} \bar{H} = \bar{J} + j\omega\varepsilon \bar{E}$$
$$\nabla . \bar{B} = 0$$
$$\nabla . \bar{D} = \rho_v$$

All field and source quantities are complex-valued vectors or scalars.

Plane Waves

Plane waves are the most fundamental solutions to the Maxwell's equations in the source free region and are also solutions of the wave equation.

The frequency-domain representation of the Wave equation is:

$$\nabla^2 \bar{E} + \omega^2 \varepsilon\mu \bar{E} = 0$$

This partial differential equation (PDE) is referred as the Helmholtz equation.

Denote, $k = \omega\sqrt{\varepsilon\mu}$, the wavenumber in (rad/m), then $\nabla^2 \bar{E} + k^2 \bar{E} = 0$

The solution to this PDE can be obtained by using the method of *separation of variables* as:

$$\bar{E}(x, y, z) = \bar{E}_o e^{-jk_x x - jk_y y - jk_z z}$$

The constants k_x, k_y and k_z determine the direction of the propagation of the wave and

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

related to the dispersion relation:

$$k^2 = \omega^2 \varepsilon \mu$$

$\bar{E}_o$, is a constant vector with complex coefficients, determining the polarization of the wave.

Also note that the wavelength is:

$$\lambda = \frac{2\pi}{k} = \frac{u}{f}$$

In free space:

$$\lambda = \frac{c}{f}$$

Example: Assume that, *x-polarized* uniform plane wave in free space has an amplitude of $2(mV/\mathrm{m})$ and propagates in the z-direction. Write the expression of the electric field intensity of this wave at 300MHz.

Solution:

Time-domain:

$$\bar{E}(z,t) = \hat{a}_x 2\cos(\omega t - kz)\ (mV/m)$$
$$\bar{E}(z,t) = \hat{a}_x 2\cos\left(2\pi X 3X10^8 t - 2\pi z\right)\ (mV/m)$$

Frequency Domain (Phasor form):

$$\bar{E}(z) = \hat{a}_x 2e^{-j2\pi z}\left(mV/m\right)$$

The magnetic field intensity:

Consider the electric field intensity $\bar{E} = E_0 e^{-jkz}\hat{a}_x$ in a simple medium characterized by $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$, $\sigma = 0$ and has an amplitude E_0. We use the phasor form of the Faraday's Induction Law to find the magnetic field intensity as:

$$\nabla X\,\bar{E} = -j\omega\mu\bar{H}$$

$$\nabla X\bar{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & \dfrac{\partial}{\partial z} \\ E_0 e^{-jkz} & 0 & 0 \end{vmatrix}$$

$$\bar{H} = \frac{k}{\omega\mu}\hat{a}_y E_0 e^{-jkz}$$

$$\frac{k}{\omega\mu} = \frac{\omega\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}{\omega\mu_r\mu_0} = \sqrt{\frac{\varepsilon_r\varepsilon_0}{\mu_r\mu_0}}$$

Denote:

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi \approx 377\Omega$$

the intrinsic impedance of free space. The intrinsic impedance of a lossless

($\sigma = 0$) dielectric medium is**:**

$$\eta = \sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

The real physical fields**:**

$$\bar{E} = \hat{a}_x E_0 \cos(\omega t - kz)$$

$$\bar{H} = \hat{a}_y \frac{E_0}{\eta} \cos(\omega t - kz)$$

We can write the following useful results related to the uniform plane waves:

1) $\bar{E}$ is $\perp$ to $\bar{H}$.
2) $|\bar{H}| = \frac{|\bar{E}|}{\eta}$
3) $\bar{E}$ and $\bar{H}$ are in phase.
4) Both $\bar{E}$ and $\bar{H}$ are $\perp$ to the direction of the propagation.

Using the following equation the magnetic field intensity can be calculated from the electric field intensity:

$$\bar{H} = \frac{\hat{n}\, X\, \bar{E}}{\eta}$$

In free space,

$$\bar{E} = E_0 e^{-jk_0 z} \hat{a}_x$$

$$k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$$

$$\bar{H} = \frac{\hat{a}_z X\, \hat{a}_x E_0 e^{-jk_0 z}}{\eta_0}$$

$$\bar{H} = \hat{a}_y \frac{E_o}{120\pi} e^{-jkz_o}\ (A/m)$$

<u>Obtaining Real-Physical Field from the Phasor Field</u>

To obtain the real physical field we multiply the phasor field by $e^{j\omega t}$ and take the real part. i.e.

$$f(x,t) = \mathrm{Re}[F(x) e^{j\omega t}]$$

The time-domain expression of the above magnetic field expression is:

$$\bar{H} = \hat{a}_y \frac{E_o}{120\pi} \mathrm{Re}\left(e^{j\omega t} e^{-jkz_o}\right)$$

$$\bar{H} = \frac{\hat{a}_y E_o \cos\left(\omega t - k_0 z\right)}{120\pi} (A/m)$$

Conducting Materials

Conducting materials are characterized by permittivity ε, permeability μ and conductivity σ. These materials are called lossy dielectric materials.

The time-domain expressions of the Maxwell's equations in a source- free lossy medium are:

$$\nabla X \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \qquad \nabla . \bar{E} = 0$$

$$\nabla X \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \qquad \nabla . \bar{H} = 0$$

The curl equations above will be useful for the simulation of the wave in a lossy medium.

The equations in the phasor forms are:

$$\nabla X \bar{E} = -j\omega\mu\bar{H} \qquad \nabla . \bar{E} = 0$$

$$\nabla X \bar{H} = \sigma \bar{E} + j\omega\varepsilon\bar{E} \qquad \nabla . \bar{H} = 0$$

We can write the Ampere's Law equation as:

$$\nabla X \bar{H} = \sigma \bar{E} + j\omega\varepsilon\bar{E} = j\omega(\varepsilon + \frac{\sigma}{j\omega})\bar{E} = j\omega\varepsilon_c \bar{E}$$

Where,

$$\varepsilon_c = \varepsilon + \frac{\sigma}{j\omega}$$

is the complex permittivity.

Loss Tangent

$$\tan\delta_c = \frac{\sigma}{\omega\varepsilon}$$

is called the *loss tangent* of the medium and it is a measure of the power loss in the medium. In practice, it is found that the loss tangent increases, for lower end of the microwave frequencies.

The following approximations are very useful for conducting materials:

The medium is said to be;

1) a good dielectric, if $\frac{\sigma}{\omega\varepsilon} << 1$
2) a good conductor, if $\frac{\sigma}{\omega\varepsilon} >> 1$.

Complex Intrinsic Impedance

Since the lossy materials have complex permittivity, the intrinsic impedance is also complex:

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}}$$

For good conductors:

$$\eta_c \approx (1+j)\sqrt{\frac{\omega\mu}{2\sigma}}$$

The x-polarized electric field intensity propagating in the + z direction will attenuate by a factor of $e^{-\alpha z}$ and will take the following form:

$$\bar{E} = \hat{a}_x E_o e^{-\alpha z} \cos(\omega t - \beta z)$$

Where α is the attenuation constant (Nepers/m) and β is the phase constant in (rad/m). (The attenuation of $1(Np/m) = 8.69(db/m)$).

For good conductors:

$$\alpha = \beta = \sqrt{\pi f \mu\sigma}$$

The phase velocity is:

$$u_p = \frac{\omega}{\beta}(m/s)$$

The wavelength is:

$$\lambda = \frac{2\pi}{\beta}$$

The Skin Depth

Skin depth is another important characteristic of a lossy dielectric. It is defined as the distance measured from the surface of the lossy medium over which the magnitude of the field is reduced to $1/e$ or approximately 37%, of the field at the surface of the medium.

For good conductors:

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

The corresponding $\bar{H}$ -field is:

$$\bar{H} = \frac{\hat{n}\,X\,\bar{E}}{\eta_c}$$

We see that, for the lossy dielectrics $\bar{E}$ and $\bar{H}$ fields are out of phase.

Propagating, Standing and Evanescent Waves

There are three basic types of waves: propagating, standing and evanescent. The plane wave discussed above represents a single wave propagating in one direction ($+z$). In a more general case, there are waves propagating in two directions. i.e. Plane waves propagating in the positive and negative (z) directions. Thus the general solution for the wave equation:

$$\nabla^2 E_x(z) + k^2 E_x(z) = 0$$

Is,

$$E_x(z) = E_0^+ e^{-jkz} + E_0^- e^{jkz}$$

A standing wave is the superposition of two plane waves of equal amplitudes propagating in the opposite directions. A standing wave is formed, when the coefficients E_0^+ , E_0^- and k are real and the wave is reflected from a conducting plate.

In certain cases, it is possible to have complex wave numbers (i.e $k = j\alpha$). Then the term e^{-jkz} becomes $e^{-\alpha z}$. If α is positive, the wave groves as z increases. Such waves are known as evanescent waves. Evanescent waves occur when a plane wave strikes on a dielectric interface at an angle larger then the incident wave or when a mode is excited in a waveguide at a frequency that is lower than the cutoff frequency.

A waveguide is a guiding structure which consists of four conducting walls. So, waveguide wave modes have the form of a standing wave in the coordinate's transverse to the axis of the waveguide.

<u>Energy and Power</u>

We would like to derive equations governing electromagnetic (EM) energy and power. Starting with Maxwell's equations:

$$\nabla X\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad (1)$$

$$\nabla X\bar{H} = \bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t} \quad (2)$$

Apply $\bar{H}.$ to the first equation and $\bar{E}.$ to the second:

$$\bar{H}.(\nabla X\bar{E} = -\frac{\partial \bar{B}}{\partial t})$$

$$\bar{E}.(\nabla X\bar{H} = \bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t})$$

Subtracting:

$$\bar{H}.(\nabla X\bar{E}) - \bar{E}.(\nabla X\bar{H}) = -\bar{H}.(\frac{\partial \bar{B}}{\partial t}) - \bar{E}.(\bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t})$$

Since:

$$\nabla.(\bar{E}X\bar{H}) = \bar{H}.(\nabla X\bar{E}) - \bar{E}.(\nabla X\bar{H})$$

$$\nabla.(\bar{E}X\bar{H}) = -\bar{H}.(\frac{\partial \bar{B}}{\partial t}) - \bar{E}.(\bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t})$$

Integration over the volume of interest:

$$\int_v \nabla.(\bar{E}X\bar{H})dv = \int_v [-\bar{H}.(\frac{\partial \bar{B}}{\partial t}) - \bar{E}.(\bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t})\ ]dv$$

Applying the divergence theorem we obtain the Poyntings Theorem:

$$\oint_s \bar{E}X\bar{H}.\hat{n}ds = -\int_v \bar{H}.\frac{\partial \bar{B}}{\partial t}dv - \int_v \bar{E}.(\bar{J}_{imp} + \bar{J}_{ind} + \frac{\partial \bar{D}}{\partial t})dv$$

Explanation of different terms:

Poynting Vector in (W/m^2):

$$\bar{P} = \bar{E} X \bar{H}$$

The power flowing out of the surface S in (W**):**

$$P_0 = \oint_s \bar{P}.\hat{n}\, ds$$

Dissipated Power:

$$P_d = \int_v (\bar{E}.\bar{J}_{ind}) dv = \int_v \sigma \bar{E}.\bar{E} dv = \int_v \sigma \left|\bar{E}\right|^2 dv$$

Supplied Power (W**):**

$$P_s = \int_v (\bar{E}.\bar{J}_{imp}) dv$$

Magnetic Power (W**):**

$$P_m = \int_v \bar{H}.\frac{\partial \bar{B}}{\partial t} dv = \int_v \mu \bar{H}.\frac{\partial \bar{H}}{\partial t} dv$$
$$= \frac{\partial}{\partial t} \int_v \frac{1}{2} \mu \left|H\right|^2 dv = \frac{\partial}{\partial t} W_m$$

W_m **: Magnetic energy**

Electric Power (W):

$$P_e = \int_v \bar{E}.\frac{\partial \bar{D}}{\partial t} = \int_v \varepsilon \bar{E}.\frac{\partial \bar{E}}{\partial t} dv$$
$$= \frac{\partial}{\partial t}\int_v \frac{1}{2}\varepsilon \left|\bar{E}\right|^2 dv = \frac{\partial}{\partial t} W_e$$

W_e**: Electric energy**

Conservation of EM energy:

$$P_0 = -P_s - P_d - \frac{\partial}{\partial t}(W_e + W_m)$$

The time-averaged power density:

$$\bar{P}_{av} = \frac{1}{2}\mathrm{Re}\left\{\bar{E}(\bar{r})X\ \bar{H}^*(\bar{r})\right\}$$