DIRECTIVITY, D

Ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D = \frac{U}{U_o} = \frac{4\pi U}{P_{rad}}$$

If the direction is not specified, it implies the direction of maximum radiation intensity:

$$D_{\max} = D_o = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{rad}}$$

Example: Compare the directivity of $U = A_o \sin\theta$ (W/Sr) and $U = A_o \sin^2\theta$ (W/Sr).

Solution:

Consider, $U = A_o \sin\theta$

$U_{\max} = A_o$ along $\theta = \dfrac{\pi}{2}$

$$\mathrm{P}_{rad} = \pi^2 A_o$$

$$D_o = \frac{4\pi U_{\max}}{\mathrm{P}_{rad}} = \frac{4}{\pi} = 1.27$$

Now consider, $U = A_o \sin^2\theta$

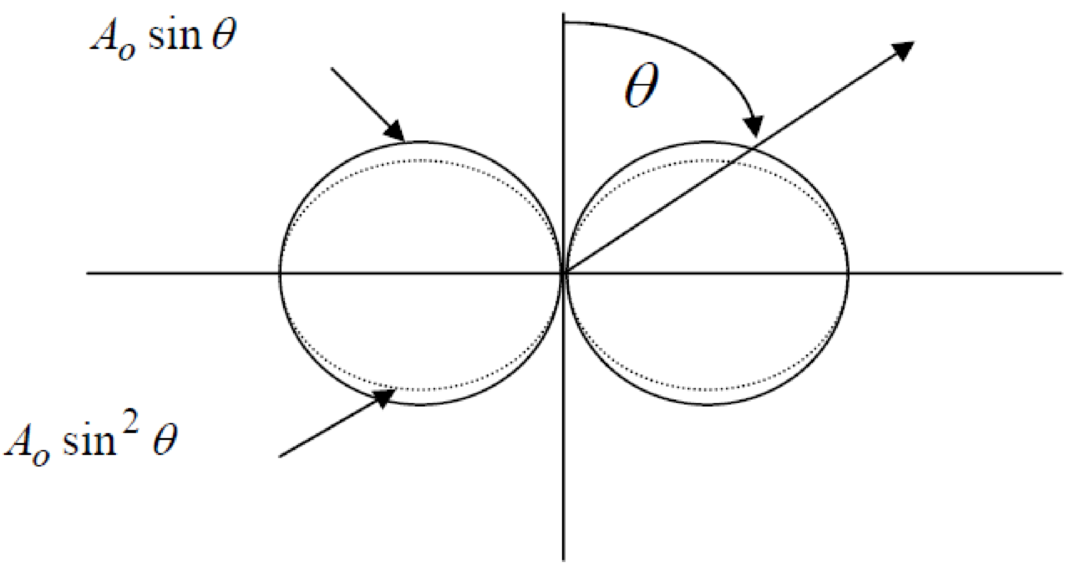
$U_{\max} = A_o$ along $\theta = \dfrac{\pi}{2}$

The total radiated power:

$$\mathrm{P}_{rad} = \left(\frac{8\pi}{3}\right) A_o$$

$$D_o = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi A_o}{\frac{8\pi}{3} A_o} = \frac{3}{2} = 1.5$$

Both patterns are omnidirectional but $U = A_o \sin^2 \theta$ has more directional characteristics (narrover) in the eleveation plane. It's direvtivity is higher compared to $U = A_o \sin\theta$ showing how good is in directing the enery in a certain direction.



The directivity of an isotropic source is unity. All other sources have maximum directivity larger then unity.

In the above examples the directivity is zero $\theta = 0^o$.

The directivity $0 < D < D_{\max}$

PARTIAL DIRECTIVITIES

The directivity:

$$D = \frac{U(\theta,\phi)}{U_O} = \frac{4\pi U_{\max}}{P_{rad}} \text{ (Dimensionless)}$$

$$D(dB) = 10\log_{10}(D)$$

For antennas with orthogonal polarization components, the radiation intensity of an antenna:

$$U \approx \frac{1}{2\eta}\left[\left|E_\theta(\theta,\phi)\right|^2 + \left|E_\phi(\theta,\phi)\right|^2\right]$$

Where,

$$U_\theta = \left|E_\theta(\theta,\phi)\right|^2$$

$$U_\phi = \left|E_\phi(\theta,\phi)\right|^2$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \left(U_\theta + U_\phi\right)\sin\theta d\theta d\phi$$

we define the partial directivities as:

$$D_\theta = \frac{4\pi U_\theta}{\left(P_{rad}\right)_\theta + \left(P_{rad}\right)_\phi}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$

$$D_o = D_\theta + D_\phi$$

U_θ : *Radiation* intensity in a given direction contained in θ field component.

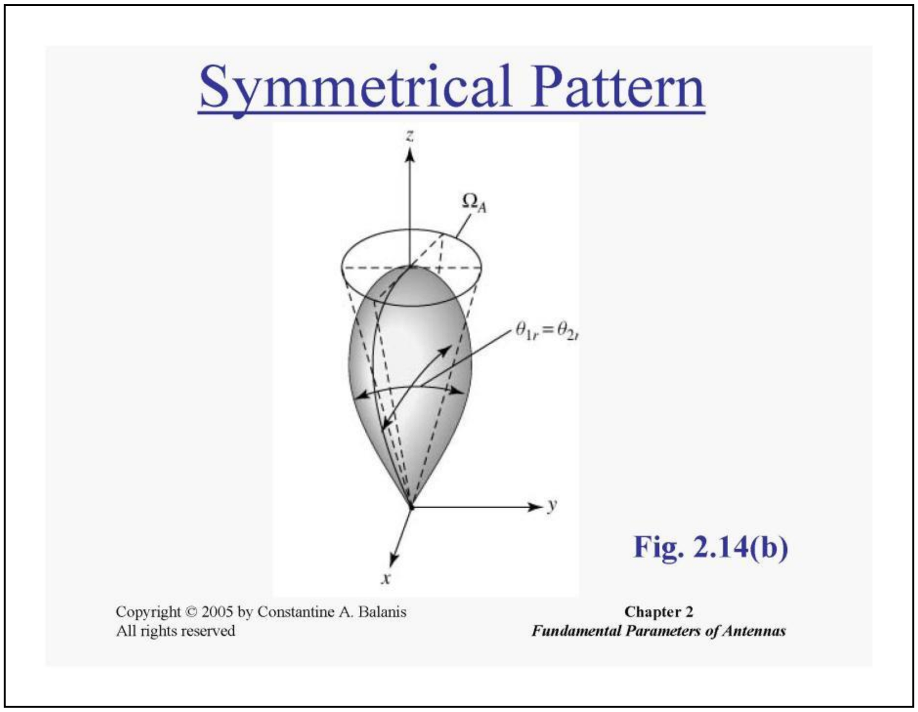
U_ϕ : *Radiation* intensity in a given direction contained in ϕ field component.

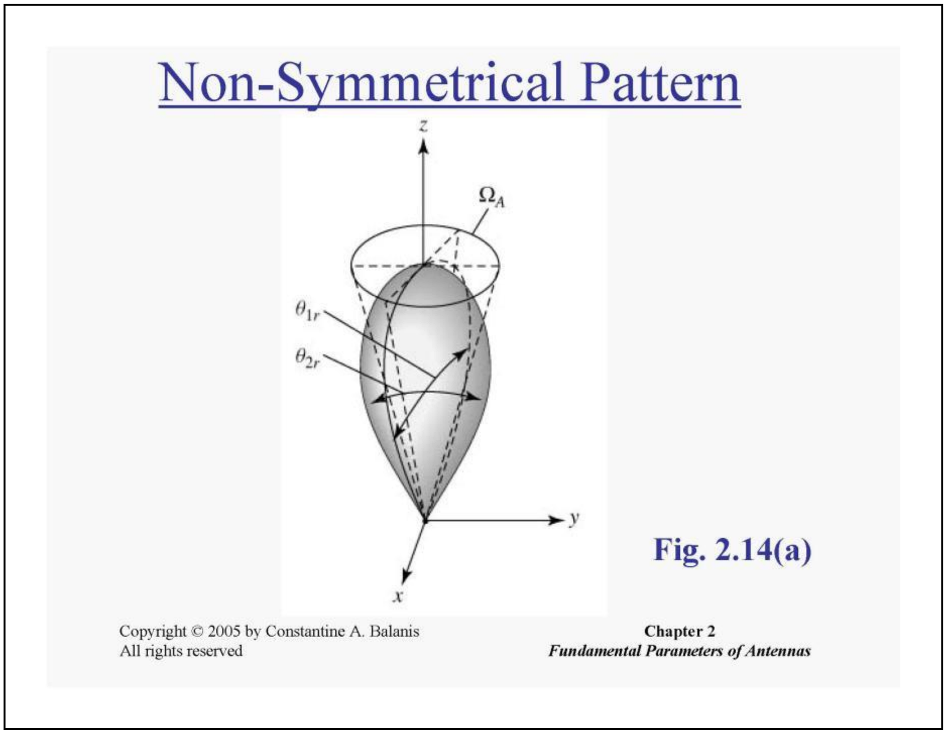
$(P_{rad})_\theta$: *Radiated* power in a given direction contained in θ field component.

$(P_{rad})_\phi$: *Radiated* power in a given direction contained in ϕ field component.

DIRECTIONAL PATTERNS

Instead of the exact expressions of the directivity, some approximated simple expressions are used to calculate the directivity.





For the antennas with one narrow major lobe and negligible minor lobes the beam solid angle can be approximated by the product of the half-power beamwidths in two perpendicular planes:

$$\Omega_A \approx \Theta_{1r}\Theta_{2r}$$

Thus,

$$D_o = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\Theta_{1r}\Theta_{2r}}$$

Where,

Θ_{1r} =Half-power beamwidth in one plane (rad).

Θ_{2r} =Half-power beamwidth in a plane at a right angle to the other (rad).

If the beamwidths are known in degrees:

$$D_o = \frac{4\pi\left(180/\pi\right)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41253}{\Theta_{1d}\Theta_{2d}}$$

Θ_{1d} =Half-power beamwidth in one plane (degees).

Θ_{2d} =Half-power beamwidth in a plane at a right angle to the other (degrees).

GAIN

Another useful measure describing the performance of an antenna is the **gain**.

Absolute gain of the antenna (in the given direction) is defined as the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna was radiated isotropically.

Mathematically:

$$\text{Gain} = 4\pi\frac{\text{radiation intensity}}{\text{total input power}} = 4\pi\frac{U\left(\theta,\phi\right)}{P_{in}} = G\left(\theta,\phi\right) \quad \text{Dimensionless}$$

P_{in} is the input power accepted by the antenna. This definition does not include losses due to mismatches of impedance or polarization.

The maximum value of gain is:

$$G = \frac{4\pi U_{\max}}{P_{in}}$$

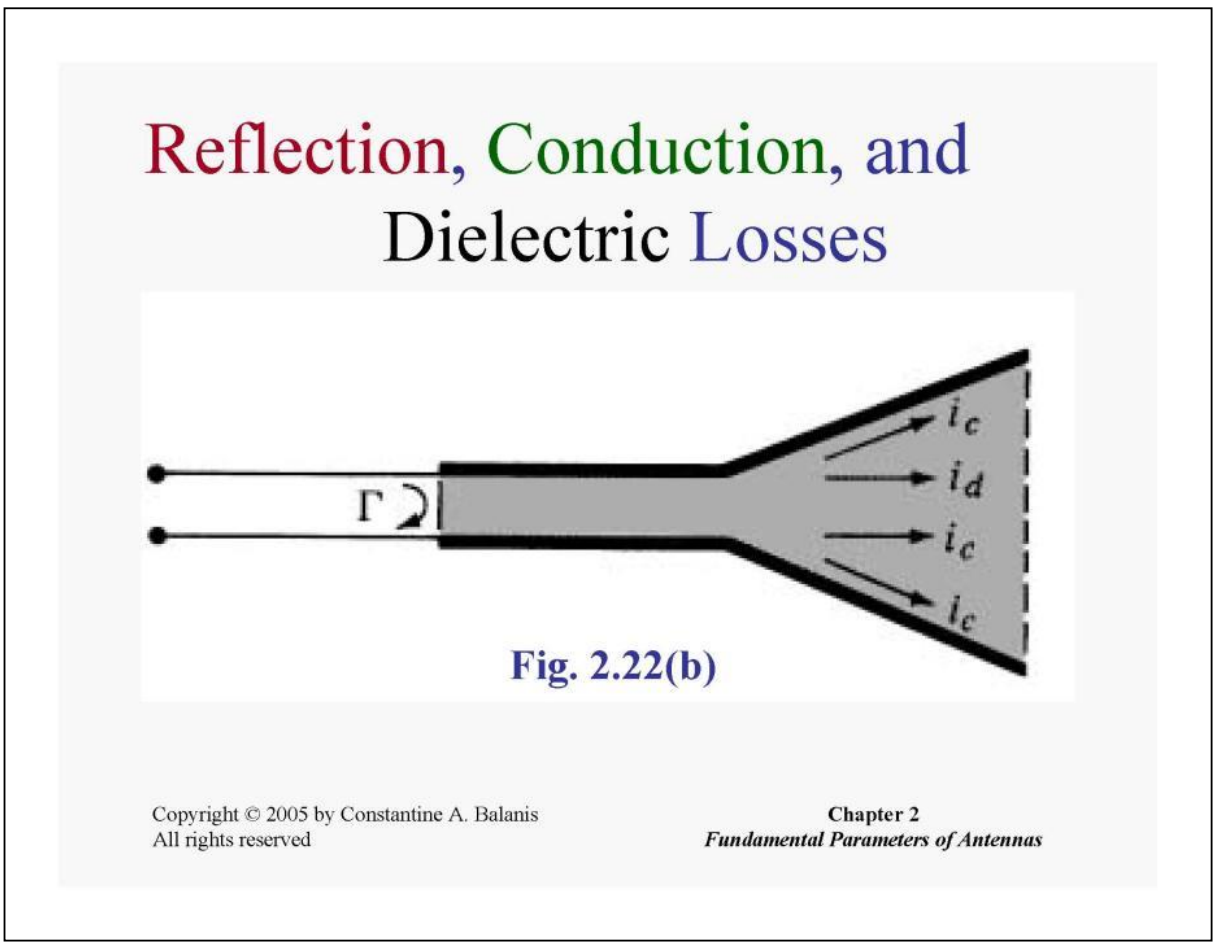
So, gain can be expressed as a function of θ and ϕ and can also be given as a value in a specific direction.

Directivity can be viewed as the gain an antenna would have if all input power appeared as the radiated power; that is $P_{in} = P$.

The portion of the input power P_{in} that does not appear as radiated power is absorbed on the antenna and nearby structures.

ANTENNA EFFICIENCY

There are number of efficiencies associated with an antenna. The total antenna efficiency e_o is used to take the losses at the input terminals and within the structure of the antenna.



1) Reflection because of the mismatch between the transmission line and the antenna.
2) I^2R losses (conduction and dicelectric).

The overall efficiency (total) can be written as:

$$e_o = e_r e_c e_d$$

Where:

$e_o =$ Total efficiency (dimensionless),

$e_r = \left(1-|\Gamma|^2\right)$ missmuch efficiency (dimensionless),

$e_c =$ conduction efficiency (dimensionless),

$e_d =$ dielectric efficiency (dimensionless),

$\Gamma =$ Voltage reflection coefficient at the input terminals of the antenna

$$\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}$$

$Z_{in} =$ Input impedance of the antenna Ω.

$Z_o =$ Characteristic impedance of the line Ω.

VSWR= Voltage Standing Wave Ratio $= \frac{1+|\Gamma|}{1-|\Gamma|}$

It is difficult to compute the conduction and dielectric losses, but they can be determined experimentally. It is more convinient to write:

$$e_0 = e_r e_{cd} = e_{cd}\left(1-|\Gamma|^2\right)$$

where

$$e_{cd} = e_c e_d$$

Radiation Efficiency e_{cd}

The antenna radiation efficiency, which is used to relate the Gain and Directivity.

The radiation efficiency is defined as:

$$e_{cd} = \frac{P_{rad}}{P_{in}}, \qquad 0 \le e_{cd} \le 1$$

The above equation takes into accout the losses of the antenna itself and does not take the losses when the antenna element is connected to the transmission line.

$$G(\theta,\phi) = e_{cd}\left[\frac{4\pi U(\theta,\phi)}{P_{rad}}\right]$$

$$G(\theta,\phi) = e_{cd}\frac{U(\theta,\phi)}{U_o}$$

$$G(\theta,\phi) = e_{cd} D(\theta,\phi)$$

$$G = e_{cd} D$$

$$G_{dB} = 10\log G$$

$$D_{dB} = 10\log D$$

The connection losses are usually refered as reflection (mismatch) losses and they are taken into accout by introducing a reflection efficiency $e_r = \left(1 - |\Gamma|^2\right)$.