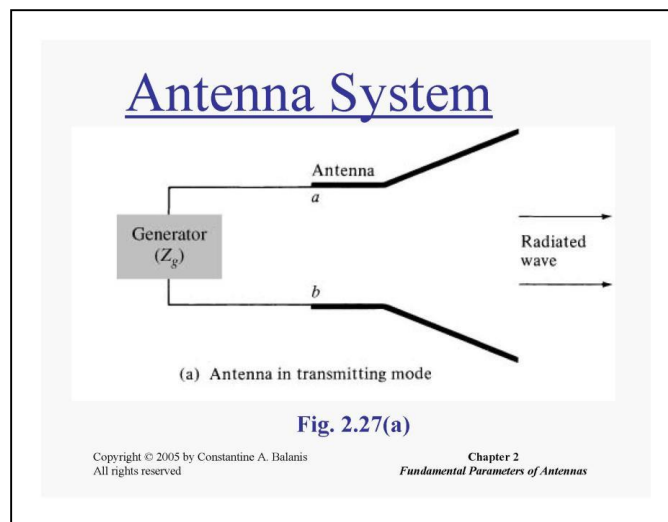


## INPUT IMPEDANCE

The input impedance of an antenna is the impedance presented by the antenna at its terminals. Thus, suitable terminals must be defined for an antenna. The ratio of the input voltage to current or the ratio of the appropriate components of the electric field to the magnetic field at these terminals will give the input impedance. These terminals are designated as **a-b** as shown in the following figure.



Assume that the antenna is isolated (i.e. the  $Z_A$  of the antenna will be affected by other antennas or objects nearby). So,

$$Z_A = R_A + jX_A$$

where

$Z_A$  = Antenna impedance at terminals a-b ohms

$R_A$  = Antenna resistance at terminals a-b ohms.

$X_A$  = Antenna reactance at terminals a-b ohms.

In general,

$R_A$  = (dissipation) radiation resistance+ ohmic losses on the antenna

$$R_A = R_r + R_L$$

According to the **reciprocity theorem** the impedance of an antenna is identical during reception and transmission.

The antenna is assumed to be attached to the generator with an impedance of:

$$Z_g = R_g + jX_g$$

$R_g$  = Resistance of the generator (ohms).

$X_g$  = Reactance of the generator (ohms).

$$\text{Radiation Efficiency} = \frac{R_r}{R_L + R_r}$$

If  $R_L = 0$  for a lossless antenna.

## ANTENNA NOISE POWER AND ANTENNA TEMPERATURE

The power radiated by the surroundings is called "NOISE". Antenna receives noise as well.

$$P_A = kT_A \Delta f$$

$T_A$  is the antenna temperature and describes how much noise an antenna produces in a given environment. This is antenna noise temperature not a physical temperature.

$k$  is the Boltzman's Constant. ( $k = 1.38 \times 10^{-23}$  J/K)

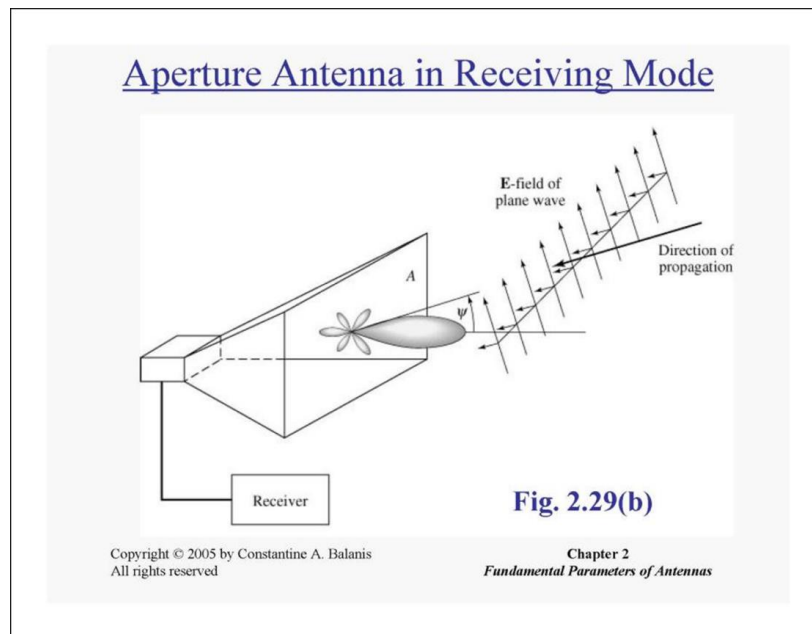
$\Delta f$  : Bandwidth (Hz)

$$T_A = \frac{\iint T_B(\theta, \phi) G(\theta, \phi) \sin \theta d\theta d\phi}{\iint_s G(\theta, \phi) \sin \theta d\theta d\phi}$$

$T_B$  : Antenna temperature (effective noise temperature of the antenna radiation resistance (K)).

## ANTENNA EFFECTIVE APERTURE

With each antenna we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. One of these areas is the **effective area**. This may be related to the surface area of the antenna but it is not equal to it.



Effective aperture is defined as the ratio of the power delivered to the load, to the incident power density:

$$A_e = \frac{P_T}{W_i}$$

where,

$A_e$  = effective area (effective aperture) ( $m^2$ ).

$P_T$  = Power delivered to the load (W).

$W_i$  = Power density of the incident wave ( $W/m^2$ ).

$A_e$ , effective aperture is directly related to the directive gain of the antenna:

$$A_e = \frac{\lambda^2}{4\pi} D_g$$

If the antenna is not polarization matched we include the PLF. If the antenna is not lossless we include  $e_t$ .

So in general:

$$A_e = \frac{\lambda^2}{4\pi} D_g \text{ PLF } e_t$$

As

$$e_t D_g = G$$

We can write:

$$A_e = \frac{\lambda^2}{4\pi} G \text{ PLF}$$

(we may be given G directly)

Maximum Power delivered to the load =  $A_{e\max} W_i$

All the power collected by the antenna is not delivered to the load.

Under the conjugate matching only half of the captured power is delivered to the load; the other half is scattered and dissipated as heat.

The aperture antenna efficiency:

$$\mathcal{E}_{ap} = \frac{A_{e\max}}{A_p} = \frac{\text{maximum effective area}}{\text{physical area}}$$

For aperture type antennas (waveguides, horns, reflectors) the maximum effective area cannot exceed the physical area but it can be equal to it.

$$A_{e\max} \leq A_p$$

Therefore the maximum aperture efficiency cannot exceed unity (100%).

$$0 \leq \epsilon_{ap} \leq 1$$

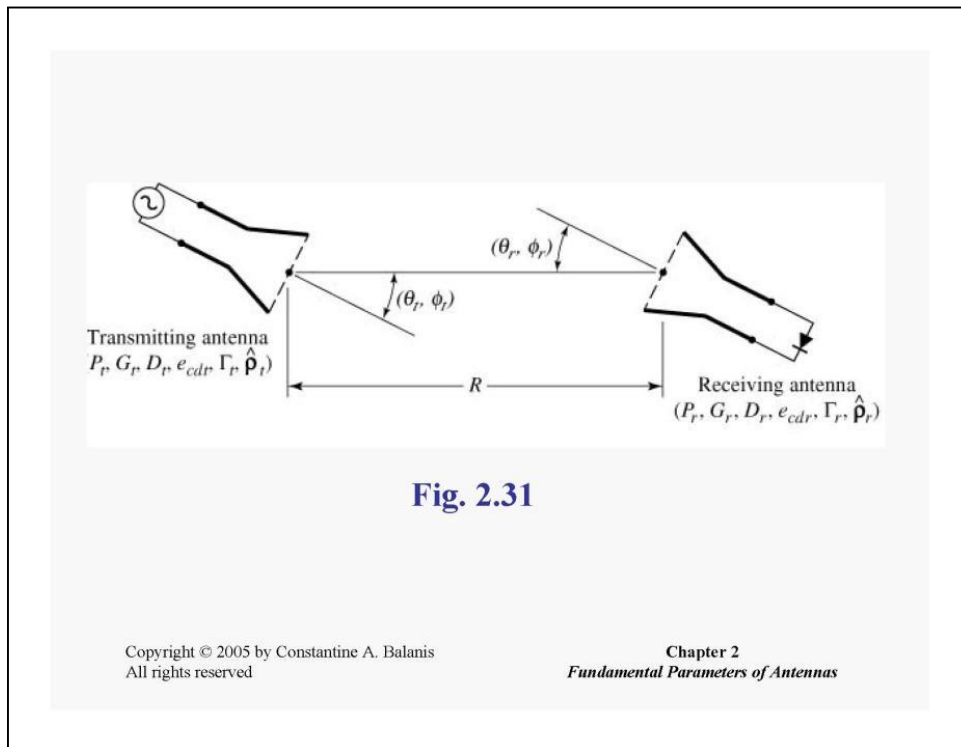
For wire antennas:

$$A_{em} \gg A_p$$

$$\epsilon_{ap} \gg 1$$

### FRIIS TRANSMISSION EQUATION

The Friis Transmission equation relates the power received and the power transmitted between two antennas separated by a distance  $R > \frac{2D^2}{\lambda}$  where  $D$ , is the maximum dimension of either antenna (two antennas must be in the far field)



First assume that the transmitting antenna is initially isotropic.

The isotropic power density  $W_o$  at a distance  $R$  from the antenna is:

$$W_o = e_t \frac{P_t}{4\pi R^2}$$

$e_t$  = radiation efficiency of the transmitting antenna.

For a non-isotropic transmitting antenna, power density in the direction  $\theta_t, \phi_t$

$$W_t = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2} = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2}$$

The effective area  $A_r$  of the receiving antenna is related to its efficiency  $e_r$  and directivity  $D_r$  by:

$$A_r = e_r D_r(\theta_r, \phi_r) \left( \frac{\lambda^2}{4\pi} \right)$$

The amount of power  $P_r$  collected by the received antenna:

$$P_r = e_r D_r(\theta_r, \phi_r) \frac{\lambda^2}{4\pi} W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) P_t}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

or,

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2} \quad \text{Friis Formula}$$

Including impedance mismatching and polarization losses:

$$\frac{P_r}{P_t} = e_{cdr} e_{cdr} (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2) \left( \frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

and

$$\frac{P_r}{P_t} = \left( \frac{\lambda}{4\pi R} \right)^2 G_t G_r$$

Atmospheric loss must be included if exists.  $\frac{P_r}{P_t}$  must be multiplied with ALF.