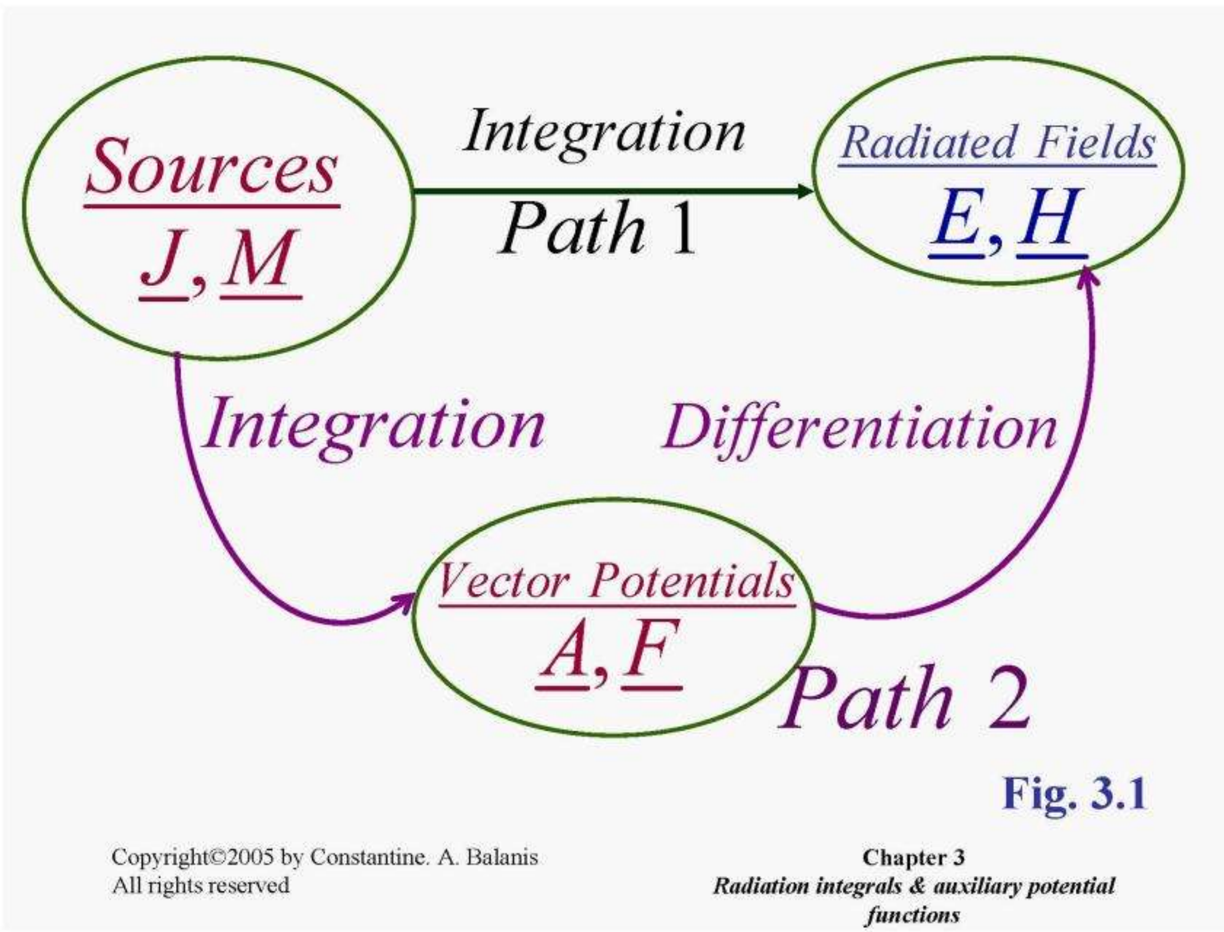
VECTOR POTENTIALS



THE VECTOR POTENTIAL $\bar{A}$

Since $\bar{B}$ is always solenoidal $\nabla.\bar{B}=0$, and the divergence of a curl is zero, we can write:

$$\nabla.\nabla X\bar{A}=0$$

We can write:

$$\bar{B}_A=\nabla X\bar{A}$$

or

$$\bar{H}_A=\frac{\nabla X\bar{A}}{\mu}$$

Substitude into the Maxwell's eqyuations:

$$\nabla X\bar{E}_A=-j\omega\mu\bar{H}_A$$

$$\nabla X\bar{E}_A=-j\omega\mu\bar{H}_A=-j\omega\nabla X\bar{A}$$

$$\nabla X\left(\bar{E}_A+j\omega\bar{A}\right)=0$$

From the vector identity:

$$\nabla X\left(-\nabla\phi_e\right)=0$$

$$\bar{E}_A+j\omega\bar{A}=-\nabla\phi_e$$

$$\bar{E}_A=-\nabla\phi_e-j\omega\bar{A}$$

Where, ϕ_e is the electric scalar potential which is a function of position.

Take the curl of both sides of:

$$\bar{H}_A = \frac{\nabla X \bar{A}}{\mu}$$

$$\nabla X \mu \bar{H}_A = \nabla X \nabla X \bar{A}$$

and use:

$$\nabla X \nabla X \bar{A} = \nabla(\nabla.\bar{A}) - \nabla^2 \bar{A}$$

$$\nabla X \mu \bar{H}_A = \nabla(\nabla.\bar{A}) - \nabla^2 \bar{A}$$

For a homogeneous:

$$\mu \nabla X \bar{H}_A = \nabla(\nabla.\bar{A}) - \nabla^2 \bar{A}$$

Equating Maxwell's equations:

$$\nabla X \bar{H}_A = \bar{J} + j\omega\varepsilon \bar{E}_A$$

So:

$$\mu \bar{J} + j\omega\mu\varepsilon \bar{E}_A = \nabla(\nabla.\bar{A}) - \nabla^2 \bar{A}$$

Substitute

$\bar{E}_A = -\nabla \phi_e - j\omega \bar{A}$ in the above equatoin, Gives:

$$\nabla^2 \overline{A} + k^2 \overline{A} = -\mu \overline{J} + \nabla\left(\nabla.\overline{A}\right) + \nabla\left(j\omega\mu\varepsilon\phi_e \right)$$

$$= -\mu \overline{J} + \nabla\left(\nabla.\overline{A} + j\omega\mu\varepsilon\phi_e\right)$$

Where $k^2 = \omega^2 \mu\varepsilon$

Let,

$$\nabla.\overline{A} + j\omega\mu\varepsilon\phi_e = 0$$

Which is known as the Lorentz condition. Then:

$$\nabla^2 \overline{A} + k^2 \overline{A} = -\mu \overline{J}$$

For a current source $\overline{J}$:

$$\overline{A} = \frac{\mu}{4\pi} \iiint_v \overline{J} \, \frac{e^{-jkR}}{R}$$